Problem 1. Find the sum of the series
\[ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n}{3^m (3^m n + 3^m m)}. \]

Problem 2. Prove that there exists a positive constant \( C \) such that for any polynomial \( P \in \mathbb{R}[x] \) of degree less than 2020, we have that
\[ P(0) \leq C \cdot \int_{-1}^{1} |P(x)| \, dx. \]

Problem 3. The sequence \( \{a_n\} \) satisfies
\[ a_1 = 1; \quad a_2 = 2; \quad a_3 = 24 \quad \text{and for } n \geq 4:\]
\[ a_n = \frac{6a_{n-1}a_{n-3}}{a_{n-2}a_{n-3}} - 8a_{n-2}a_{n-3}. \]
Prove that for each positive integer \( n \), we have that \( a_n \) is an integer multiple of \( n \).

Problem 4. Let \( P \in \mathbb{C}[x] \) be a polynomial of degree \( n \) such that \( P(x) = Q(x) \cdot P''(x) \), where \( Q(x) \) is a quadratic polynomial and \( P'' \) is the double derivative of \( P \). Show that if \( P(x) \) has at least two distinct roots, then it must have \( n \) distinct roots.