PROBABILISTIC APPROACHES TO SYMMETRISED MANY-PARTICLE SYSTEMS

STEFAN ADAMS

We consider a symmetrised functional of Brownian bridges, which is related to the partition function of an interacting many-particle system. The aim is to calculate the associated free energy in the thermodynamic limit where temperature and particle density remain positive. More precisely, we give a lower and upper bound on the free energy in terms of certain variational principles. The bounds coincide if either temperature is fixed and the particle density is small or conversely if the density remains fixed and the temperature is high. The novel idea is a representation of the partition function in terms of a marked point process, where the marks are Brownian bridges starting and ending at the corresponding points of the point process. Based on the large deviations results for marked point processes, we employ an argument analogously to Varadhan's lemma to eventually obtain the bounds on the partition function. The difference in upper and lower bounds, however, is not merely technical, but hints at the emergence of Bose-Einstein condensation, where infinitely long cycles appear and whose description remains a major challenge of the field. In a second part we outline the connection to random permutation and random partitions models.