PARTIAL MATCH QUERIES IN RANDOM QUADTREES

Imagine that points are falling uniformly on the unit square according to a Poisson process. At each time a point falls in a rectangle, it splits this rectangle into four subrectangles according to its horizontal and vertical coordinates. This process is called the uniform quadtree structure (see Fig.below).

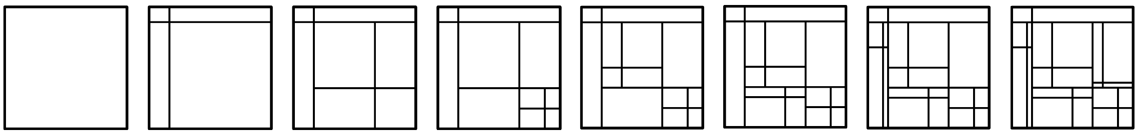

Figure 1: The first 7 splittings of a quadtree.

We are interested in the so-called partial match query. Equivalently, for $x \in [0,1]$, we focus on the number $N_t(x)$ of rectangles in the quadtree at time t whose horizontal coordinate intersects x. This quantity has been studied for a long time since the work of Flajolet, Gonnet, Puech and Robson in were that showed that $N_t(x)$ should be roughly of order t^β where $\beta = (\sqrt{17}-3)/2$. In a very recent breakthrough, Broutin, Neininger and Sulzbach used the "contraction method" to obtain a convergence in distribution of the rescaled processes $\{t^{-\beta}N_t(x) : 0 \le x \le 1\}$ towards a random continuous process $\{\tilde{M}_\infty(x) : 0 \le x \le 1\}$ characterized by a recursive decomposition. We will show that this convergence actually holds in a stronger sense:

Theorem 1. *For every $x \in [0,1]$ we have the following* almost sure *convergence*

$$t^{-\beta}N_t(x) \xrightarrow[t\to\infty]{a.s.} K_0 \cdot \tilde{M}_\infty(x).$$

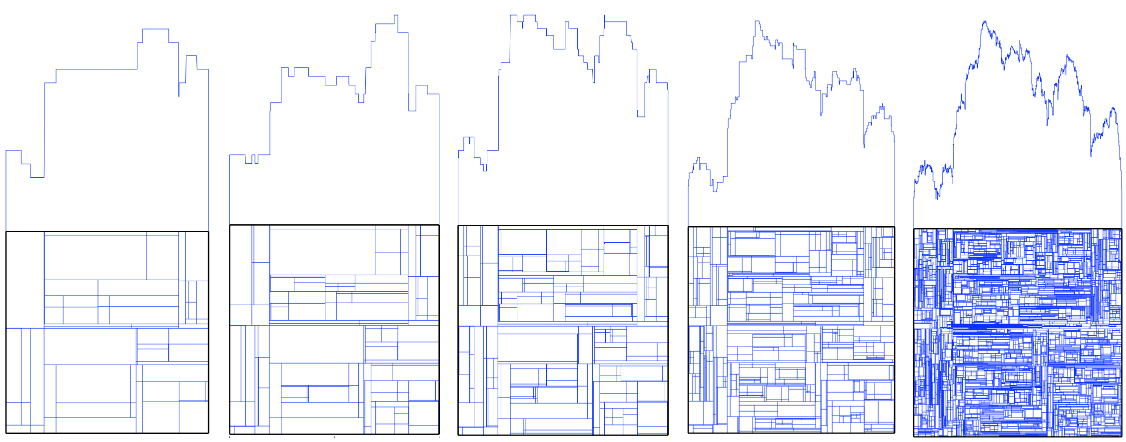

Figure 2: An illustration of the strong convergence of the partial match queries. The curves above the quadtree represent the renormalized processes $t^{-\beta}(N_t(x))_{x\in[0,1]}$ for $t = 20, 50, 100, 500$ and 3000.

Our approach shares many similarities with the theory of self-similar fragmentations.