Quiz 1

2017-09-21

Last name ....

First name ....

Student number ....

Email ....

Grade

For each computation of limits in this test, if the limit does not exist, indicate whether it diverges to $-\infty$ or $+\infty$.

1. Each part of this question is worth 1 mark.

(a) **(1 pt)** Compute

$$\lim_{x\to -3} \frac{x^2+7}{|x+1|}$$

Solution. As $x \to -3$, the numerator converges to $(-3)^2 + 7 = 16$, while the denominator converges to $|-3+1| = |-2| = 2$. So, the limit equals 8.

(b) **(1 pt)** Compute

$$\lim_{x\to +\infty} \frac{3x^2-2}{2x^2+x+5}$$

Solution. We compute

$$\lim_{x\to +\infty} \frac{3x^2-2}{2x^2+x+5} = \lim_{x\to +\infty} \frac{\frac{3x^2-2}{x^2}}{\frac{2x^2+x+5}{x^2}} = \lim_{x\to +\infty} \frac{3-\frac{2}{x^2}}{2+\frac{1}{x}+\frac{5}{x^2}} = \frac{3}{2}$$

2. Each part of this question is worth 2 marks. **You have to show all your work in order to get credit.**

(a) **(2 pt)** Compute

$$\lim_{x \to 1^-} \frac{\sqrt{(x-1)^2}}{x^2 - 1}$$

Solution. As $x \to 1^-$, we have that $1 - x$ is positive and so, $\sqrt{(x-1)^2} = \sqrt{(1-x)^2} = 1 - x$. Thus we get

$$\lim_{x \to 1^-} \frac{\sqrt{(x-1)^2}}{x^2 - 1} = \lim_{x \to 1^-} \frac{1-x}{(x-1)(x+1)} = \lim_{x \to 1^-} \frac{-1}{x+1} = -\frac{1}{2}.$$

(b) **(2 pt)** Compute

$$\lim_{x \to -\infty} x - \sqrt{x^2 + x}$$

Solution. As $x \to -\infty$, we have a limit type of "$-\infty - \infty$"; so, the above limit does not exist and it diverges to $-\infty$.

3. This question is worth 4 marks. **You have to show all your work in order to get credit.**

Find the two real numbers a and b such that $\lim_{x\to 2} f(x)$ exists for

$$f(x) = \begin{cases} \frac{x^2-ax-6}{x-2} & \text{if} \quad x < 2 \\ 3+bx & \text{if} \quad x > 2. \end{cases}$$

Solution.

First of all, the limit exists if

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x).$$

Since the denominator of $\frac{x^2-ax-6}{x-2}$ converges to 0 as $x \to 2$, then we need the numerator also to converge to 0, i.e., $\lim_{x\to 2^-} x^2 - ax - 6 = 0$ and so, $4 - 2a - 6 = 0$, i.e., $\underline{a = -1.}$ With this value $a = -1$, we have

$$\begin{aligned} &\lim_{x\to 2^-} f(x) \\ &= \lim_{x\to 2^-} \frac{x^2+x-6}{x-2} = \lim_{x\to 2^-} \frac{(x+3)(x-2)}{x-2} = \lim_{x\to 2^-} (x+3) = 5. \end{aligned}$$

So, we require $\lim_{x\to 2^+} f(x) = 5$. Then, we see

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 3 + bx = 3 + 2b.$$

Thus, $3 + 2b = 5$, therefore $\underline{b = 1.}$

1. Each part of this question is worth 1 mark.

(a) **(1 pt)** Compute

$$\lim_{x\to -1} \frac{3}{\sqrt{7x^3+11}}$$

Solution.

$$\lim_{x\to -1} \frac{3}{\sqrt{7x^3+11}} = \lim_{x\to -1} \frac{3}{\sqrt{7(-1)^3+11}} = \frac{3}{\sqrt{4}} = \frac{3}{2}.$$

(b) **(1 pt)** Compute

$$\lim_{t\to -\infty} \frac{t^2+4t}{5t^2+1}$$

Solution. We compute

$$\lim_{t\to -\infty} \frac{t^2+4t}{5t^2+1} = \lim_{t\to -\infty} \frac{t^2(1+4/t)}{t^2(5+1/t^2)} = \lim_{t\to -\infty} \frac{1+4/t}{5+1/t^2} = \frac{1}{5}$$

2. Each part of this question is worth 2 marks. **You have to show all your work in order to get credit.**

(a) **(2 pt)** Compute

$$\lim_{x\to 1^+} \frac{|1-x|}{x^2+x-2}$$

Solution. As $x \to 1^+$, we have that $x-1$ is positive and so, $|1-x| = |x-1| = x-1$. Thus we get

$$\lim_{x\to 1^+} \frac{|1-x|}{x^2+x-2} = \lim_{x\to 1^+} \frac{x-1}{(x-1)(x+2)} = \lim_{x\to 1^+} \frac{1}{x+2} = \frac{1}{3}.$$

(b) **(2 pt)** Compute

$$\lim_{x\to -\infty} x^2 + x$$

Solution. As $x \to -\infty$, we have a limit type of "$\infty - \infty$", however if we factor out the leading term, we have

$$\lim_{x\to -\infty} x^2 + x = \lim_{x\to -\infty} x^2 \left(1 + \frac{1}{x}\right) = \infty$$

for $\left(1 + \frac{1}{x}\right) \to 1$ and $x^2 \to \infty$ as $x \to -\infty$.

3. This question is worth 4 marks. **You have to show all your work in order to get credit.**

Find the two real numbers a and b such that $\lim_{x\to -2} f(x)$ exists for

$$f(x) = \begin{cases} \dfrac{x^2 + ax - 4}{x+2}, & x < -2 \\ -x^2 + bx, & x > -2 \end{cases}$$

Solution.

First of all, the limit exists if

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x).$$

For the fraction $\frac{x^2+ax-4}{x+2}$, the denominator converges to 0 as $x \to -2$, then we need the numerator also to converge to 0, i.e., $\lim_{x\to -2^-} x^2+ax-4 = 0$ and so, $(-2)^2 - 2a - 4 = 0$, i.e., $\underline{a = 0.}$ With this value $a = 0$, we have

$$\begin{aligned} &\lim_{x\to -2^-} f(x) \\ &= \lim_{x\to -2^-} \frac{x^2 + ax - 2^2}{x+2} = \lim_{x\to -2^-} \frac{x^2 - 2^2}{x+2} \\ &= \lim_{x\to -2^-} \frac{(x+2)(x-2)}{x+2} = \lim_{x\to -2^-} x - 2 = -2 - 2 = -4. \end{aligned}$$

So, we require $\lim_{x\to -2^+} f(x) = -4$. Then, we see

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} -x^2 + bx = -4 - 2b$$

Therefore $\underline{b = 0.}$

1. Each part of this question is worth 1 mark.

(a) **(1 pt)** Compute

$$\lim_{x\to 2} \frac{x^2-10}{|x-5|}$$

Solution. As $x \to 2$, the numerator converges to $(2)^2 - 10 = -6$, while the denominator converges to $|2-5| = |-3| = 3$. So, the limit equals -2.

(b) **(1 pt)** Compute

$$\lim_{x\to+\infty} \frac{2x^2-5}{3x^2-3x-2}$$

Solution. We compute

$$\lim_{x\to+\infty} \frac{2x^2-5}{3x^2-3x-2} = \lim_{x\to+\infty} \frac{\frac{2x^2-5}{x^2}}{\frac{3x^2-3x-2}{x^2}} = \lim_{x\to+\infty} \frac{2-\frac{5}{x^2}}{3-\frac{3}{x}-\frac{2}{x^2}} = \frac{2}{3}$$

2. Each part of this question is worth 2 marks. **You have to show all your work in order to get credit.**

(a) **(2 pt)** Compute

$$\lim_{x \to -2^-} \frac{\sqrt{(x+2)^2}}{x^2-4}$$

Solution. As $x \to -2^-$, we have that $x+2$ is negative and so, $\sqrt{(x+2)^2} = \sqrt{(-x-2)^2} = -x-2$. Thus we get

$$\lim_{x \to -2^-} \frac{\sqrt{(x+2)^2}}{x^2-4} = \lim_{x \to -2^-} \frac{-x-2}{(x+2)(x-2)} = \lim_{x \to -2^-} \frac{-1}{x-2} = \frac{1}{4}.$$

(b) **(2 pt)** Compute

$$\lim_{x \to -\infty} 2x - \sqrt{x^2 - 3x}$$

Solution. As $x \to -\infty$, we have a limit type of "$-\infty - \infty$"; so, the above limit does not exist since it diverges to $-\infty$.

3. This question is worth 4 marks. **You have to show all your work in order to get credit.**

Find the two real numbers a and b such that $\lim_{x\to 1} f(x)$ exists for

$$f(x) = \begin{cases} \dfrac{x^2 - ax - 6}{x-1} & \text{if} \quad x < 1 \\ 3 + bx & \text{if} \quad x > 1. \end{cases}$$

Solution.

First of all, the limit exists if

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x).$$

Since the denominator converges to 0 as $x \to 1$, then we need the numerator also to converge to 0, i.e., $\lim_{x\to 1^-} x^2 - ax - 6 = 0$ and so, $1 - a - 6 = 0$, i.e., $\underline{a = -5.}$ With this value $a = -5$, we have

$$\lim_{x\to 1^-} f(x)$$
$$= \lim_{x\to 1^-} \frac{x^2+5x-6}{x-1} = \lim_{x\to 1^-} \frac{(x+6)(x-1)}{x-1} = \lim_{x\to 1^-} (x+6) = 7.$$

So, we require $\lim_{x\to 1^+} f(x) = 7$. Then, we see

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 3 + bx = 3 + b.$$

Thus, $3 + b = 7$, therefore $\underline{b = 4.}$