Syllabus Math 423/502, January-April 2016

Rough course schedule, subject to later adjustments:

Week 1 Jan. 3-7:

- Outline.
- Representations of associative algebras, definition and examples; example of a group algebra and representation of a group; quotient representation; generators of a representation; direct sum of representations; subrepresentations; irreducible, indecomposable representations semisimple representations; Schur's Lemma; irreducible finite dimensional representations of a commutative algebra over an algebraically closed field; subrepresentations in semisimple representations; socle of a representation;

Week 2 Jan. 10-14:

- Jordan-Hölder and Krull-Schmidt Theorem; radical of a finite dimensional algebra. Artin-Wedderburn theorem.
- Representations of a finite group; examples; Maschke's theorem;

  References: For the first two weeks of the course, refer to Etingof's notes (Chapter 1 and 2) `http://math.mit.edu/~etingof/replect.pdf` as well as the first 2 chapters of Webb's book (given in the references for the class), in particular Lemma 1.2.2, Prop. 1.2.3 and Corollary 1.2.5.

Week 3 Jan. 17-21: Simple representations of a finite group over $\mathbb{C}$:

- Character table: definition and examples.
- Orthogonality relations and consequences.
- The number of irreducible/simple representations;
  References: Chapter 3, 3.1-3.4 of Webb's book (see the examples therein).

Week 4 Jan. 24-28: Representations of a finite group over $\mathbb{C}$ (2):

Chapter 4.2 and 4.3 of Webb's book. Etingof's notes Chapter 4.

- Induction and restriction, adjunctions.
  More on Frobenius pairs and their role in representation theory (and topology etc): `http://arxiv.org/pdf/1008.5084.pdf` Categorifications from planar diagrammatics, M. Khovanov.
- Complex representations of $\mathrm{GL}_2(\mathbb{F}_q)$.
  Recommended reading: notes by Garrett `http://www.math.umn.edu/~garrett/m/v/toy_GL2.pdf`
  See also the following books:
  Representation Theory by Fulton and Harris, and The Local Langlands Conjecture for GL(2), by Bushnell and Henniart.
  To read more about the general philosophy of the Langlands conjectures: An elementary introduction to the Langlands program, by Gelbart (available online).

Week 5 Feb. 1-5: Interlude on categories and functors.

Week 6 Feb. 8-12: Feb. 8, Family Day. Review. Midterm Exam on the 10th or the 12th

Feb. 15-19: Winter Break

Week 7 Feb. 22-26: Exact functors, the functor $V \mapsto V^H$ where V is a representation of G and H is a subgroup of G (in different characteristics), Projective modules, Principal indecomposable modules of a finite dimensional algebra. Idempotents.

Week 8 Feb. 29-Mar. 4: K-theory of an Abelian Category. Ext^* between modules over an algebra. Snake lemma and long exact sequences. Examples. Characterization of projectives (resp. injectives) in terms of Ext^*.

Week 9 Mar. 7-11: Definition of group cohomology. $H^1(G,k) = Hom(G_{ab},(k,+))$. Example of the unipotent subgroup of $\mathrm{GL}_2(\mathbb{F}_q)$. Computation of $Ext^*_G(k,k)$ where G is a cyclic group.

Week 10 Mar. 14-18:

Week 11 Mar. 21-25: Review; Midterm Exam **Good Friday**.

Week 12 Mar. 28-Apr. 1: **Easter Monday.**

Week 13 Apr. 4-8: