*This midterm has **3 questions** on **6 pages**, for a total of 30 points.*

*Duration: 50 minutes*

- Write your name or your student number on **every** page.
- **You need to show enough work to justify your answers.**
- Continue on the **back of the previous page** if you run out of space. You also have extra space at the end of the booklet.
- You have to turn in **ALL 6 SHEETS** of this booklet even if you don't use all of them.
- This is a closed-book examination. **None of the following are allowed**: documents or electronic devices of any kind (including calculators, cell phones, etc.)

LAST name: ______________________

First name: (including all middle names): ______________________

Student Number: ______________________

Signature: ______________________

Circle the name of your instructor: Rachel Ollivier Justin Tzou

| Question: | 1 | 2 | 3 | Total |
|---|---|---|---|---|
| Points: | 10 | 10 | 10 | 30 |
| Score: | | | | |

We recall that for a vector field $\mathbf{F}$ in $\mathbb{R}^3$, we have:

$$\mathbf{curl}(\mathbf{F}) = \nabla \times \mathbf{F}$$

$$\text{div}(\mathbf{F}) = \nabla \cdot \mathbf{F}$$

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1. True or False. Circle the right answer. You do not need to justify your answer. No partial credit.

2 marks (a) Let $\mathbf{F} = (P, Q, R)$ be a vector field in $\mathbb{R}^3$ and assume that $P$, $Q$ and $R$ have continuous second order partial derivatives. Then $\text{div}(\mathbf{curl}(\mathbf{F})) = 0$.

True False

2 marks (b) The line integral of the vector field $\langle yz, xz, xy\rangle$ along the triangle with vertices $(1, 0, 0)$, $(1, -1, 0)$ and $(1, 1, 0)$, traversed in that order, is equal to 0.

True False

2 marks (c) There exists a vector field $\mathbf{G}$ in $\mathbb{R}^3$ such that $\mathbf{curl}(\mathbf{G}) = \langle x + yz, y + zx, z + xy\rangle$.

True False

2 marks (d) If the curve $C$ is oriented positively and bounds the surface $S$ in the $x - y$ plane, then $\frac{1}{2}\oint_C xdx + ydy$ is equal to the area of $S$.

True False

2 marks (e) The line integral of the vector field $\frac{1}{x^2+y^2}\langle -y, x\rangle$ along the circle $x^2 + y^2 = 1$ oriented counterclockwise is zero.

True False

2. Let $\mathbf{F} = \langle 3, x, y^2 \rangle$ and $S$ be the surface $z = x^2 + y^2$ with $0 < z < 4$.

1 mark (a) Compute $\mathbf{curl}(\mathbf{F}) = \nabla \times \mathbf{F}$.

2 marks (b) Give a parametrization of the surface $S$. Do not forget the domain of definition.

3 marks (c) Compute the flux of $\mathbf{curl}(\mathbf{F})$ through $S$ with the upward orientation for $S$, i.e., compute the double integral

$$\iint_S \mathbf{curl}(\mathbf{F}) \cdot d\mathbf{S}.$$

2 marks (d) Consider the 2-radius circle $C$ centered at $(0, 0, 4)$ and contained in the plane $z = 4$ traversed counterclockwise when seen from above. What is the value of the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ ? Justify your answer.

2 marks (e) Let $S'$ be the disk $x^2 + y^2 < 4$ with $z = 4$. We choose the downward orientation for $S'$. What is the flux of $\mathbf{curl}(\mathbf{F})$ through $S'$ ? Justify your answer.

3. We consider the plane with equation $x + 2y + z = 5$ and a simple closed curve $C$ on this plane. Suppose that $C$ is oriented counterclockwise when viewed from above. We call $\mathcal{A}$ the area of the surface enclosed by $C$. Let $\mathbf{F} = \langle \frac{z^2}{4}, \frac{x^2}{2}, y^2 \rangle$.

7 marks

(a) Give the expression of the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ as a function of $\mathcal{A}$.

3 marks

(b) Compute the line integral $\oint_{C_1} \mathbf{F} \cdot d\mathbf{r}$, where $C_1$ is the the curve consisting of the three line segments, $L_1$ from $(0, 0, 5)$ to $(0, \frac{5}{2}, 0)$, then $L_2$ from $(0, \frac{5}{2}, 0)$ to $(5, 0, 0)$, and finally $L_3$ from $(5, 0, 0)$ to $(0, 0, 5)$.