This midterm has ***4 questions*** *on* ***6 pages****, for a total of 40 points.*

Duration: 50 minutes

- Write your name or your student number on **every** page.
- **You need to show enough work to justify your answers.**
- Continue on the **back of the previous page** if you run out of space. You also have extra space at the end of the booklet.
- You have to turn in **ALL 6 SHEETS** of this booklet even if you don't use all of them.
- This is a closed-book examination. **None of the following are allowed**: documents or electronic devices of any kind (including calculators, cell phones, etc.)

LAST name: ______________________________

First name: (including all middle names): ______________________________

Student Number: ______________________________

Signature: ______________________________

Circle the name of your instructor: Rachel Ollivier Justin Tzou

Question:	1	2	3	4	Total
Points:	13	7	7	13	40
Score:					

7 marks

1. (a) Calculate the work done by the force field $\mathbf{F} = (-ay, bx, -cz)$ on a particle that moves on a path C, where C is composed of

   i. a straight line from $(0,-1,0)$ to $(0,1,0)$,

   ii. straight lines from $(0,-1,0)$ to $(1,-1,0)$ to $(0,1,0)$.

3 marks

(b) What is the condition on a, b, and c for $\mathbf{F}$ to be conservative ? Hint: you may use part (a). (Be careful, you have to prove that the condition you find is necessary AND sufficient.)

3 marks

(c) For $a = 2$, $b = -2$ and $c = 1$, evaluate the line integral of $\mathbf{F}$ along the curve C given by $\mathbf{r}(t) = (\ln(1+t^5), e^{t^7}, t)$ with $0 \le t \le 1$.

7 marks 2. Compute the line integral of the vector field

$$\mathbf{F}(x, y) = (ye^{xy}, xe^{xy} + x)$$

along the curve $x(t) = \cos(t)$, $y(t) = 4\sin(t)$ where $0 \leq t \leq 2\pi$. Hint: decompose $\mathbf{F}$ as a sum of two vectors fields, and justify that one of them is conservative.

7 marks 3. Calculate the arc length of the curve parameterized by

$$x(t) = \frac{1}{2}t + \frac{1}{2}\cos t \sin t\,, \quad y(t) = \frac{1}{2}\sin^2 t\,; \qquad 0 \le t \le \pi\,.$$

Suggestion: you may use the formulas $\sin(2t) = 2\sin(t)\cos(t)$ and then $1 + \cos(2t) = 2\cos^2(t)$ to simplify the calculations.

4. Consider the curve C given by $\mathbf{r}(t) = e^{-t}\cos(t)\mathbf{i} + e^{-t}\sin(t)\mathbf{j} + \sqrt{2}e^{-t}\mathbf{k}$.

3 marks (a) Let $L(t)$ denote the arc length of the curve from the point $(1, 0, \sqrt{2})$ to the point with parameter t. Compute $\lim_{t\to+\infty} L(t)$.

1 mark (b) Compute the unit tangent vector $\mathbf{T}(t)$.

1 mark (c) Compute the unit normal vector $\mathbf{N}(t)$ and check that $\mathbf{N}(0) = (-\sqrt{2}/2, -\sqrt{2}/2, 0)$.

3 marks (d) Give the coordinates of the center of the osculating circle at parameter $t = 0$. We recall that the curvature at parameter t is given by $\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$ and that the osculating circle at parameter t has radius $1/\kappa(t)$.

4 marks (e) Give the equation of the osculating plane at parameter $t = 0$.

1 mark (f) Is C a plane curve?

Extra space. Continue your work here.