Worksheet 6

We recall that given a square matrix A
– an *eigenvalue* for A is a real number λ such that there exists a **nonzero** vector $\mathbf{x}$ satisfying $A\mathbf{x} = \lambda\mathbf{x}$
– the vector $\mathbf{x}$ is then called an *eigenvector* for A and the value λ
– the set of all eigenvectors for the value λ (= the set of solutions of $A\mathbf{x} = \lambda\mathbf{x}$) is called the *eigenspace* of A for the value λ.

We saw that λ is an eigenvalue for A if and only if it is a root for the characteristic polynomial

$$p_A = \det(A - xI)$$

that is to say a solution to the equation $\det(A - xI) = 0$.

If λ is an eigenvalue for A, then $(x - \lambda)$ is a factor of the polynomial p_A. The power of $(x - \lambda)$ in the decomposition of $\det(A - xI)$ is called the *multiplicity* of the eigenvalue λ.

Problem 1. Find the eigenvalues of the following matrices and compute the multiplicity and eigenspace for each of the eigenvalues.

$$B = \begin{pmatrix} 0 & -2 \\ -4 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} 9 & 3 & -5 \\ -2 & 1 & 2 \\ 5 & 3 & -1 \end{pmatrix}, \quad E = \begin{pmatrix} -1 & 4 & 2 \\ -1 & 4 & 0 \\ -1 & 1 & 3 \end{pmatrix}$$

On the examples above we observe that

$$\boxed{\text{Multiplicity of the eigenvalue } \lambda} \quad \ldots\ldots \quad \boxed{\text{Dimension of the eigenspace for } \lambda}$$

Problem 2. Consider the linear transformation T_A attached to the matrix $A = \begin{pmatrix} -4/5 & 3/5 \\ 3/5 & 4/5 \end{pmatrix}$.

(1) Find the eigenvalues of A and the corresponding eigenspaces.
(2) Show that there is a basis $\mathcal{B}$ of $\mathbb{R}^2$ made of eigenvectors for A.
(3) What is T_A ?
(4) Let P be the matrix whose columns are the vectors of $\mathcal{B}$. Compute $P^{-1}AP$.