Worksheet 5

Problem 1. What is the rank of the following matrices?

$$A = \begin{pmatrix} 2 & 3 & 1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 0 & 3 \\ 1 & 0 & 0 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 & 0 & 2 \\ 1 & 2 & 0 & 3 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 & 1 \\ 9 & 2 & 1 \\ 0 & 2 & 3 \end{pmatrix}$$

Give a basis for their null space and their image (the image of a matrix is the same as the range of the corresponding linear map, or the *column space* as they call it in the book).

Problem 2. Let $n \geq 3$. Is the following size n matrix invertible?

$$M = \begin{pmatrix} 1 & 1 & 0 & 0 & \dots \\ 0 & 1 & 1 & 0 & \dots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \dots & 0 & 1 & 1 \\ 1 & 0 & \dots & 0 & 1 \end{pmatrix}$$

Problem 3. Compute the determinant of

$$D = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix}.$$