Worksheet 4

Given a $m \times n$ matrix A, and the corresponding linear transformation

$$T_A : \mathbb{R}^n \to \mathbb{R}^m$$

we recall :

- the Rank of A is the dimension of the Range of T_A, i.e the dimension of the vector space spanned by the columns of A.
- the Null space of A, also called the Kernel of T_A, is the set of $\mathbf{x} \in \mathbb{R}^n$ such that $A\mathbf{x} = \mathbf{0}$.

We admit the following formula (which you have to know) :

$$\boxed{n = \ \text{Rank}(A) + \ \dim(\text{Null}(A))}$$

Problem 1. Let

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 0 & 2 & 2 \\ 1 & 2 & 1 \end{pmatrix}.$$

(1) Find a basis of the Null space of A.
(2) What is the rank of A ?
(3) Show that the Range of T_A is the plane with equation $2x + y - 4z = 0$ (this is harder than an exam question).
(4) Find a basis of the Range of T_A.

Problem 2. Let A be a square matrix. Show that $\boxed{A \text{ is invertible}} \Leftrightarrow \boxed{\text{Null}(A) = \{0\}} \Leftrightarrow \boxed{\text{Rank}(A) = n}$

Problem 3. (1) Show that the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 2 & 6 \end{pmatrix}$ is invertible.

(2) Compute its determinant.