Worksheet 1

Problem 1. Among the following ones, which families of vectors are independent ?

(1) $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{u}_2 = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$

(2) $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{u}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{u}_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.

(3) $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{u}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{u}_3 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

Problem 2. (1) Let $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $\mathbf{v}_3 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. Show that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly dependent (that it to say not independent).

(2) Let $\mathbf{v}_1, \dots, \mathbf{v}_m$ a family of m vectors in $\mathbb{R}^n$ and suppose that $m > n$. Show that the family is linearly dependent.

Problem 3. Let $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Is Span$\{\mathbf{v}_1, \mathbf{v}_2\}$ the line with equation $y = 3x+4$?

Problem 4. (1) Let $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{u}_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{u}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

Show that Span$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is the plane with equation $x + y = z$.

(2) Let $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{u}_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{u}_3 = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$.

Show that Span$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is $\mathbb{R}^3$.

(3) Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 5 \end{pmatrix}$

Is the system $A\mathbf{x} = \begin{pmatrix} 1 \\ 5 \\ 10 \end{pmatrix}$ consistent ?

(4) Let $B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$

Is the system $B\mathbf{x} = \begin{pmatrix} 1 \\ 5 \\ 10 \end{pmatrix}$ consistent ?