Math 121 Practice Problem Set 1 for the second midterm
(Based on Chapter 8 and Sections 9.1–9.4)

1. Determine whether the sequence
$$a_1 > \sqrt{2}, \quad a_{n+1} = \frac{a_n}{2} + \frac{1}{a_n}, \quad n = 1, 2, 3 \cdots$$
has a limit. If it does, then find the limit.
2. Do the following series
$$\sum_{n=10}^{\infty} \frac{(-1)^{n-1}}{\ln \ln n} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{n^2 \cos(n\pi)}{1+n^3}$$
converge absolutely, converge conditionally or diverge?
3. Find the area of the region inside the curve $r^2 = 2\cos 2\theta$ and outside $r = 1$.
4. Identify the curve whose polar equation is given by $r = \sec\theta \tan\theta$.
5. Find the intersections of the pair of curves $r = \theta$, $r = \theta + \pi$.
6. Find the volume of the solid obtained by rotating about the x-axis the region bounded by that axis and one arch of the cycloid $x = at - a\sin t$, $y = a - a\cos t$.
7. Does the alternating series test continue to hold if the assumption
$$|a_{n+1}| \leq |a_n| \quad \text{for all } n \geq N$$
is dropped? Prove this statement if it is true, or give a counterexample.