Math 121 Assignment 9

Due Friday April 9

■ Practice problems:

- Try out as many problems from Sections 9.5–9.6 as you can, with special attention to the ones marked as challenging problems. As a test of your understanding of the material, work out the problems given in the chapter review. You may skip the ones that require computer aid.

■ Problems to turn in:

1. Find the centre, radius and interval of cnvergence of each of the following power series.

$$\text{(a)} \sum_{n=0}^{\infty} \frac{1+5^n}{n!} x^n \qquad \text{(b)} \sum_{n=1}^{\infty} \frac{(4x-1)^n}{n^n}.$$

2. Expand
(a) $1/x^2$ in powers of $x+2$.
(b) $x^3/(1-2x^2)$ in powers of x.
(c) e^{2x+3} in powers of $x+1$.
(d) $\sin x - \cos x$ about $\frac{\pi}{4}$.
For each expansion above, determine the interval on which the representation is valid.

3. Find the sums of the following numerical series.

$$\text{(a)} \sum_{n=0}^{\infty} \frac{(n+1)^2}{\pi^n} \quad \text{(b)} \sum_{n=1}^{\infty} \frac{(-1)^n n(n+1)}{2^n} \quad \text{(c)} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n2^n},$$

(d) $x^3 - \dfrac{x^9}{3! \times 4} + \dfrac{x^{15}}{5! \times 16} - \dfrac{x^{21}}{7! \times 64} + \dfrac{x^{27}}{9! \times 256} - \cdots$

(e) $1 + \dfrac{x^2}{3!} + \dfrac{x^4}{5!} + \dfrac{x^6}{7!} + \dfrac{x^8}{9!} + \cdots$

(f) $1 + \dfrac{1}{2 \times 2!} + \dfrac{1}{4 \times 3!} + \dfrac{1}{8 \times 4!} + \cdots$

4. This problem outlines a srategy for verifying whether a function f is real-analytic. Recall the nth order Taylor polynomial of f centred at c:

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k,$$

and set $E_n = f(x) - P_n(x)$.

(a) Use mathematical induction to show that

$$E_n(x) = \frac{1}{n!} \int_c^x (x-t)^n f^{(n+1)}(t)\, dt,$$

provided $f^{(n+1)}$ exists on an interval containing c and x. The formula above is known as Taylor's formula with integral remainder.

(b) Use Taylor's formula with integral remainder to prove that $\ln(1+x)$ is real analytic at $x = 0$; more precisely, that the Maclaurin series of $\ln(1+x)$ converges to $\ln(1+x)$ for $-1 < x \leq 1$.