Math 121 Assignment 6
Due Friday March 12

■ Practice problems:

- Try out as many problems from Sections 7.5, 7.6, 7.8, 7.9 as you can, with special attention to the ones marked as challenging problems. As a test of your understanding of the material, work out the problems given in the chapter review. You may skip the ones that require computer aid.

■ Problems to turn in:

1. Find the centroid of the infinitely long spike shaped region lying between the x-axis and the curve $y = (x+1)^{-3}$ and to the right of the y-axis.
2. Find the centroid of the solid obtained by rotating the plane region $0 \leq y \leq 2x - x^2$ about the line $y = -2$.
3. A pyramid with a square base, 4 m on each side and four equilateral triangular faces, sits on the level bottom of a lake at a place where the lake is 10 m deep. Find the total force of the water on each of the triangular fcaes.
4. If 100 N.cm of work must be done to compress an elastic spring to 3 cm shorter than its normal length, how much work must be done to compress it 1 cm further? Recall that by Hooke's law, the force required to compress an elastic spring to x units shorter than its natural length is proportional to x.
5. A bucket is raised vertically from ground level at a constant speed of 2m/min by a winch. If the bucket weighs 1 kg and contains 15 kg of water when it starts up but loses water by leakage at the rate of 1 kg/min thereafter, how much work must be done by the winch to raise the bucket to a height of 10m?
6. The number X of metres of cable produced any day by a cable-making company is a normally distributed random variable with mean 5000 and standard deviation 200. On what fraction of the operating days will the number of metres of cable produced exceed 5,500?
7. A spinner is made with a scale from 0 to 1. Over time it suffers from wear and tends to stick at the number 1/4. Suppose it sticks at 1/4 half the time and the rest of the time it gives values uniformly distributed in the interval $[0, 1]$. What is the mean and standard deviation of the spinner's values?

8. Find the family of curves each of which intersects all the hyperbolas $xy = C$ at right angles.
9. Find a function $y(x)$ that obeys the equation $y(x) = 3 + \int_0^x e^{-y(t)}\, dt$.
10. The equation of motion of an object of mass m falling from rest near the surface of the earth with air resistance is given by

$$m\frac{dv}{dt} = mg - kv,$$

where $v = v(t)$ is the velocity of the object at time t, and g is the acceleration due to gravity near the surface of the earth. Find the velocity $v(t)$ as a function of t, and compute the limit of $v(t)$ as $t \to \infty$.