Math 121 Formula Sheet

1. **Area as limit of Riemann sums**:
   The area R lying under the graph $y = f(x)$ of a non-negative continuous function f between the vertical lines $x = a$ and $x = b$ is given by
   $$R = \lim_{\substack{n\to\infty \\ \max \Delta x_i \to 0}} \sum_{i=1}^{n} f(x_i)\Delta x_i,$$
   where $a = x_0 < x_1 < \cdots < x_{n-1} < x_n < b$ is a partition of $[a, b]$ and $\Delta x_i = x_i - x_{i-1}$.
2. **Trapezoid Rule**
   The n-subinterval trapezoid rule approximation to $\int_a^b f(x)\,dx$, denotes T_n, is given by
   $$T_n = h\left(\frac{y_0}{2} + y_1 + \cdots + y_{n-1} + \frac{y_n}{2}\right), \qquad \text{where } y_i = f(x_i).$$
   If f has a continuous second derivative on the interval $[a, b]$ satisfying $|f''(x)| \le K$ there, then the error in applying trapezoid rule is at most $K(b-a)^3/(12n^2)$.
3. **Midpoint Rule**
   If $h = (b-a)/n$, let $m_j = a + (j - \frac{1}{2})h$ for $i \le j \le n$. The midpoint rule approximation to $\int_a^b f(x)\,dx$, denoted M_n, is given by
   $$M_n = h\sum_{j=1}^{n} f(m_j).$$
   If f has a continuous second derivative on the interval $[a, b]$ satisfying $|f''(x)| \le K$ there, then the error in applying midpoint rule is at most $K(b-a)^3/(24n^2)$.
4. **Simpson's Rule**
   The Simpson's rule approximation to $\int_a^b f(x)\,dx$ based on a subdivision of $[a, b]$ into an even number n of subintervals of equal length $h = (b-a)/n$ is denotes S_n and is given by
   $$S_n = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right).$$
   If f has a continuous fourth derivative on the interval $[a, b]$ satisfying $|f^{(4)}(x)| \le K$ there, then the error in applying Simpson's rule is at most $K(b-a)^5/(180n^4)$.
5. **Pappus's Theorem**
   (a) If a plane region R lies on one side of a line L in that plane and is rotated about L to generate a solid of revolution, then the volume V of that solid is given by
   $$V = 2\pi\bar{r}A,$$
   where A is the area of R and $\bar{r}$ is the perpendicular distance from the centroid of R to L.

(b) If a plane curve C lies on one side of a line L in that plane and is rotated about that line to generate a surface of revolution, then the area S of that surface is given by

$$S = 2\pi \bar{r} s,$$

where s is the length of the curve C, $\bar{r}$ is the perpendicular distance from the centroid of C to the line L.

6. **The general normal distribution**

A random variable X on $(-\infty, \infty)$ is said to be normally distributed with mean μ and standard deviation σ (where μ is any real number and $\sigma > 0$) if its probability density function $f_{\mu,\sigma}$ is given by

$$f_{\mu,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

7. **Taylor polynomials and remainder**

If the $(n+1)$st derivative of f exists on an interval containing c and x and if

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

is the Taylor polynomial of degree n for f about $x = c$, then

$$f(x) = P_n(x) + E_n(x),$$

where

$$E_n(x) = \frac{1}{n!} \int_c^x (x-t)^n f^{(n+1)}(t)\, dt.$$

8. **Fourier series**

If $f(t)$ is a period function with fundamental period T, is continuous with a piecewise continuous derivative, then for every t,

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos(n\omega t) + b_n \sin(n\omega t)), \quad \omega = \frac{2\pi}{T},$$

where

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos(n\omega t)\, dt \quad n = 0, 1, 2, \cdots,$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin(n\omega t)\, dt, n = 1, 2, 3 \cdots.$$