# Math 100, Lecture 12, 15/2/2024

**Last time:** Manipulating Taylor expansions:

(1) Can add, multiply expansions:

to expand $f+g$, $fg$ to $n$'th order about $a$:

(1) expand $f, g$ to $n$'th order about $a$;

(2) add / multiply expansions

(3) truncate to $n$'th order

(2) To expand $\frac{1}{f}$ expand $f \approx \underbrace{C_0}_{f(a)} + C_1(x-a) + \dots + C_n(x-a)^n$

set $\frac{1}{f} \approx \frac{1}{f(a)} \frac{1}{1 + \underbrace{\frac{C_1}{f(a)}(x-a) + \frac{C_2}{f(a)}(x-a)^2 + \dots}_{-V}}$

Use geom series $\frac{1}{1-V} = 1 + V + V^2 + \dots$

(3) To expand $f(g(x))$ about $x=a$ to $n$'th order

(1) expand $g$ at $a$ to $n$'th order

set:

$$g(x) \approx g(a) + C_1(x-a) + \dots + C_n(x-a)^n$$

(2) Expand $f(u)$ about $u = g(a)$

(3) Plug in $u = g(x)$.

Example: $\frac{1}{1+x^3} = \frac{1}{1-(-x^3)}$ about $x=0$

$\downarrow$

$\sim 1 + (-x^3) + (-x^3)^2 + (-x^3)^3 + \dots$

$= 1 - x^3 + x^6 - x^9 + \dots$

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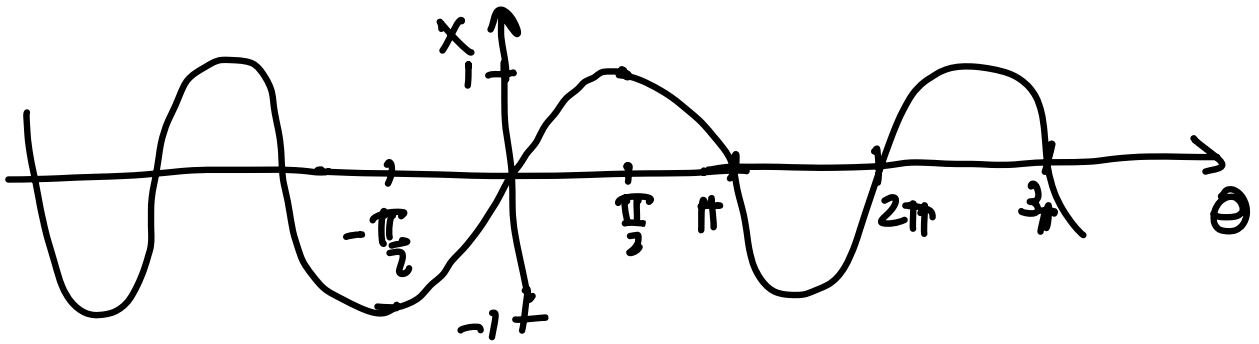
## Inverse trig functions

Familiar with function $x = \sin\theta$




inverse sine is reverse function $\theta = \arcsin(x)$

values sine are in $[-1, 1]$ so $\arcsin x$ only meaningful for $x \in [-1, 1]$

Def: $\arcsin x$ is the angle $\theta$ s.t. $\begin{cases} \sin\theta = x \\ \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}] \end{cases}$

$\arccos x$ is the angle $\theta$ s.t. $\begin{cases} \cos\theta = x \\ \theta \in [0, \pi] \end{cases}$

$\arctan x$ is " " " " $\begin{cases} \tan\theta = x \\ \theta \in (-\frac{\pi}{2}, \frac{\pi}{2}) \end{cases}$

Sometimes people write $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$

$\arcsin x \neq \frac{1}{\sin x}$