Math 100C – WORKSHEET 9
EULER'S METHOD

1. Compound interest (Bernoulli 1683)

(1) Suppose you have a \$100 bank balance which earns an annual interest rate of 30%.
   (a) Suppose the interest is paid once, at the end of the year. How much would your balance be at that time?

   (b) Suppose instead that interest is paid four times a year. What is the quarterly interest *rate*? What would the balance be at the end of the first quarter?

   (c) Suppose further that interest is *compounded*: after every quarter the interest is added to the balance. What would be the balance at the end of the year?

   (d) Suppose instead that interest is compounded *daily* and that at a particular day the balance is y dollars. What is the balance the next day?

(2) Suppose interest is compounded *continuously* and that at a particular time y the balance is $y(t)$ dollars, where t is measured in years.
   (a) What is the approximate interest rate for the period between times $t, t+h$ if h is very small?

   (b) What is the balance at time $t+h$?

- Rearranging and taking the limit $t \to 0$ we obtain the ODE $y'(t) = 30\%y(t)$. In general if the interest rate is r we discover that $y(t) = y(0)e^{rt}$.

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2. Further examples

From now on let the interest rate by r.

(3) Suppose that in addition to the interest we also have a constant income stream of b dollars per month.
   (a) What differential equation expresses our bank balance now?

   (b) What is the general solution (hint: use an ansatz of the form $Ce^{rt} + B$). What is the solution that has $y(0) = y_0$?

(4) Suppose instead that our income stream is seasonal, so that the differential equation is $y' = ry + b\sin(2\pi t)$. Find the general solution and the solution satisfying $y(0) = y_0$ using an Ansatz of the form $Ae^{rt} + B\sin(2\pi t) + C\cos(2\pi t)$.

(5) (For numerical discussion) Suppose instead the *interest rate* is seasonal, so the equation is $y' = (r + a\cos(2\pi t))y$. Can you find a solution? What if $y' = (r + a\sin(2\pi t))y + b$?