Math 100C – WORKSHEET 4
ARITHMETIC OF THE DERIVATIVE

1. Review of the derivative

(1) Expand $f(x+h)$ to linear order in h for the following functions and read the derivative off:
   (a) $f(x) = bx$

   (b) $g(x) = ax^2$

   (c) $h(x) = ax^2 + bx$.

   (d) $i(x) = \frac{1}{b+x}$

   (e) $j(x) = 4x^4 + 5x$ (hint: use the known linear approximation to $2x^2$)

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2. Arithmetic of derivatives

Fact. $(af+bg)' = af' + bg'$, $\quad (fg)' = f'g + fg'$, $\quad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
$\frac{d}{dx}x^n = nx^{n-1}$.

(2) Differentiate

(a) $f(x) = 6x^\pi + 2x^e - x^{7/2}$

(b) (Final, 2016) $g(x) = x^2 e^x$ (and then also $x^a e^x$)

(c) (Final, 2016) $h(x) = \frac{x^2+3}{2x-1}$

(d) $\frac{x^2+A}{\sqrt{x}}$

(3) Let $f(x) = \frac{x}{\sqrt{x}+A}$. Given that $f'(4) = \frac{3}{16}$, give a quadratic equation for A.

(4) Suppose that $f(1) = 1$, $g(1) = 2$, $f'(1) = 3$, $g'(1) = 4$.
   (a) What are the linear approximations to f and g at $x = 1$? Use them to find the linear approximation to fg at $x = 1$.

   (b) Find $(fg)'(1)$ and $\left(\frac{f}{g}\right)'(1)$.

(5) Evaluate
   (a) $(x \cdot x)'$ and $(x') \cdot (x')$. What did we learn?

   (b) $\left(\frac{x}{x}\right)'$ and $\frac{(x')}{(x')}$. What did we learn?

(6) The *Lennart–Jones potential* $V(r) = \epsilon\left(\left(\frac{R}{r}\right)^{12} - 2\left(\frac{R}{r}\right)^{6}\right)$ models the electrostatic potential energy of a diatomic molecule. Here $r > 0$ is the distance between the atoms and $\epsilon, R > 0$ are constants.

(a) What are the asymptotics of $V(r)$ as $r \to 0$ and as $r \to \infty$?

(b) Sketch a plot of $V(r)$.

(c) Find the derivative $\frac{dV}{dr}(r) =$

(d) Where is $V(r)$ increasing? decreasing? Find its minimum location and value.