Math 100C – WORKSHEET 1
EXPRESSIONS AND ASYMPTOTICS

1. Asymptotics: simple expressions

(1) Classify the following functions into *power laws / power functions* and *exponentials*: x^3, πx^{102}, e^{2x}, $c\sqrt{x}$, $-\frac{8}{x}$, 7^x, $8 \cdot 2^x$, $-\frac{1}{\sqrt{3}} \cdot \frac{1}{2^x}$, $\frac{9}{x^{7/2}}$, x^e, π^x, $\frac{A}{x^b}$.

(2) How does the each expression behave when x is large? small? what is x is large but negative? Sketch a plot

(a) $7 + x^2 + x^4$

(b) $x^3 - x^5$

(c) $e^x - x^4$

(d) Wages in some country grow at 2% a year (so the wage of a typical worker has the form $A \cdot (1.02)^t$ where t is measured in years and A is the wage today). The cost of healthcare grows at 4% a year (so the healthcare costs of a typical worker have the form $B \cdot (1.04)^t$ where B is the cost today). Suppose that today's workers can afford their healthcare (A is much bigger than B). Will that be always true? Why or why not?

(e) Three strains of a contagion are spreading in a population, spreading at rates 1.05, 1.1, and 0.98 respectively. The total number of cases at time t behaves like

$$A \cdot 1.05^t + B \cdot 1.1^t + C \cdot 0.98^t \,.$$

(A, B, C are constants). Which strain dominates eventually? What would the number of infected people look like?

Date: 14/9/2022, Worksheet by Lior Silberman.

2. Asymptotics of complicated expresisons

(3) Construct parse trees for the following expressions:
   (a) $e^{|x-5|^3}$
   (b) $\frac{e^x+A\sin x}{e^x-x^2}$
   (c) $\frac{1+x}{1+2x-x^2}$
   (d) $\left(\frac{t+\pi}{t-\pi}\right)\sin\left(\frac{t+\pi}{2}\right)$

(4) For each of the functions in (a),(b),(c),(d) use the parse tree to determine its asymptotics as $x \to 0$ and as $x \to \infty$.
   (a) $\left(\frac{t+\pi}{t-\pi}\right)\sin\left(\frac{t+\pi}{2}\right)$