Midterm review

$$\lim_{x\to-\infty}\sqrt{4x^2-3x}+2x+1 = \lim_{x\to\infty}\sqrt{4x^2-3x}-2x+1 = \lim_{x\to\infty} x\left(\sqrt{4-3/x}-2+\frac{1}{x}\right)$$
$$= \lim_{x\to\infty} x\cdot 0 = 0$$

1. Evaluate the following limits:

   (a) $\lim_{x\to-\infty}\sqrt{4x^2-3x}+2x+1$

$$\lim_{x\to-\infty}\sqrt{4x^2-3x}+2x+1 = \sqrt{4x^2-3x}+2x+1\cdot\frac{\sqrt{4x^2-3x}-2x-1}{\sqrt{4x^2-3x}-2x-1} = \frac{4x^2-3x-2x^2-1}{\sqrt{4x^2-3x}-2x+1} =$$
$$= \frac{2x^2-3x-1}{2x\sqrt{1-\frac{3}{4x}}-2x-1} = \lim_{x\to-\infty}\frac{x^2}{x}\cdot\frac{2-\frac{3}{x}-1/x^2}{2\sqrt{1-\frac{3}{4x}}-2-\frac{1}{x}} = \frac{2-0-0}{2\sqrt{1-0}-2} = \frac{2}{2-2} = \frac{2}{0} \text{ DNE.}$$

   (b) $\lim_{t\to-3}\frac{2t+6}{\sqrt{t+4}-1}$

$$\lim_{t\to-3}\frac{2t+6}{\sqrt{t+4}-1} = \lim_{t\to-3}\frac{2t+6}{\sqrt{t+4}-1}\cdot\frac{\sqrt{t+4}+1}{\sqrt{t+4}+1} = \lim_{t\to-3}\frac{(2t+6)\sqrt{t+4}+1}{t+4-1}$$
$$\xrightarrow{t\to3^+}\infty \qquad \xrightarrow{t\to3^-}-\infty$$

2. Show that there is a number c such that $\tan(c) = c+1$.

Let $f(x) = \tan(x) - x + 1$. Then $f(0) = 1$, $f(\pi) = 0 - \pi + 1 = -(\pi - 1) < 0$
Thus f has a zero between $0, \pi$.

3. Differentiate

   (a) $(3+x)^{\frac{3}{x}}$ $(x > 3)$

$$\log(3+x)^{3/x} = \frac{\log 3}{\log x}\log(3+x) \quad \text{so } \left((3+x)^{3/x}\right)' = -\frac{\log 3}{(\log x)^2}\log(3+x) + \frac{\log 3}{(3+x)\log x}$$

   (b) $\sin x\cos(x^2+x)$

$$\frac{d}{dx}\left(\sin x\cos(x^2+x)\right) = -\cos x\,\sin(x^2+x)\,(2x+2)$$

5. A population of algae decays exponentially.

   (a) If the population falls by a factor of 3 every 30 days, find the time needed for the population to be divided by 2.

$N = N_0 e^{-kt}$ $\quad N(30) = \frac{2}{3}N_0$ so $\frac{2}{3} = e^{-k30}$ so $\frac{\log 2}{\log 3} = -k\cdot 30$ so $k = -\frac{\log 3}{30\log 2}$

so $N(t) = \frac{1}{2}N_0$ when $\frac{1}{2} = e^{-kt}$ $\Rightarrow \frac{1}{2} = e^{\frac{\log 3}{\log 2\cdot 30}t}$ $\Rightarrow \log\frac{1}{2} = \frac{\log 3}{\log 2}\frac{t}{30}$

   (b) If the initial population is 100, what is the population after 10 days?

$\Rightarrow t = 30\cdot\frac{\log\frac{1}{2}\log 2}{\log 3}$.

$$N(10) = e^{\frac{\log 3}{\log 2}\cdot\frac{10}{30}} = e^{\frac{\log 3}{3\log 2}}.$$