Math 100 – WORKSHEET 18
THE MVT AND CURVE SKETCHING

1. The shape of a the graph

(1) Side exercise: Let f be twice differentiable on $[a, b]$.
   (a) Suppose first that $f(a) = f(b) = 0$ and that f is positive somewhere between a, b. Show that there is c between a, b so that $f''(c) < 0$.
   (b) Now let $f(a), f(b)$ take any values, but suppose $f''(x) > 0$ on (a, b). Let $L : y = mx + n$ be the line through $(a, f(a)), (b, f(b))$. Applying part (a) to $g(x) = f(x) - (mx + n)$ show that the graph of f lies below the line L.

Definition. We say f is *concave up* on an interval $[a, b]$ if its graph lies under the secant lines in this interval (equivalently: above the tangent lines). This is true if $f'' > 0$ on (a, b). We say f is *concave down* on the interval if its graph lies below the scant lines (equivalently: above the tangent lines), in particular when $f'' < 0$ on (a, b). We say that f has an *inflection point* at x_0 if its second derivative changes sign there.

(2) For each of the following functions determine its domain, and where it is increasing or decreasing. Except in part (b) also determine where the function is concave up/down.
   (a) $f(x) = e^x$
   (b) $f(x) = \frac{x-1}{x^2+1}$
   (c) $f(x) = x \log x - 2x$
   (d) $\frac{x^2-9}{x^2+3}$. You may use that $f'(x) = \frac{24x}{(x^2+3)^2}$ and that $f''(x) = 72\frac{1-x^2}{(x^2+3)^3}$.

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