

Math 100 – WORKSHEET 5

THE IVT

1. THE INTERMEDIATE VALUE THEOREM

Theorem. Let $f(x)$ be continuous for $a \leq x \leq b$. Then $f(x)$ takes every value between $f(a), f(b)$.

Checklist:

- (1) Construct function (if needed)
- (2) Check continuity
- (3) Find points of positivity & negativity
- (4) Invoke IVT

(1) Show that $f(x) = 2x^3 - 5x + 1$ has a zero in $0 \leq x \leq 1$.

(2) (Final 2011) Let $y = f(x)$ be continuous with domain $[0, 1]$ and range in $[3, 5]$. Show the line $y = 2x + 3$ intersects the graph of $y = f(x)$ at least once.

(3) $\sin x = x + 1$ has a solution.

(4) (Final 2015) Show that the equation $2x^2 - 3 + \sin x + \cos x = 0$ has at least two solutions.

(5) (Final 2018) Let g be a continuous function such that

$$\frac{x}{2} \leq g(x) \leq \frac{x}{2} + 1$$

for each positive real number x . Let $f(x) = g(x) + \sin x$. Show that there are infinitely many real numbers c such that $f(c) = \frac{c+1}{2}$.

2. DEFINITION OF THE DERIVATIVE

Definition. $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

(6) Find $f'(a)$ if

(a) $f(x) = x^2$, $a = 3$.

(b) $f(x) = \frac{1}{x}$, any a .

(c) $f(x) = x^3 - 2x$, any a . (you may use $(a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$).

(7) Express the limit as a derivative: $\lim_{h \rightarrow 0} \frac{\cos(5+h) - \cos 5}{h}$.