Math 101 – WORKSHEET 33
TAYLOR SERIES AND DERIVATIVES

1. Manipulating power series: summing series

(1) Find $\sum_{n=1}^{\infty} \frac{1}{n2^n}$.

(2) Avatars of geometric series.
   (a) Evaluate $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

   (b) Express $\sum_{n=1}^{\infty} n^2 x^n$ as a *rational function* (ratio of polynomials).

(3) Find a simple formula for $\sum_{n=0}^{\infty} \frac{e^{nx}}{n!}$.

Date: 29/3/2017, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

2. Taylor Series

The *Taylor series* of $f(x)$ *centered at* c is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n \,.$$

(4) Find the MacLaurin ($c = 0$) series of $f(x) = e^x$.

(5) (Final 2014) Find the Taylor series $g(x) = \log x$ centered at $a = 2$, as well as its radius of convergence.

(6) (Final 2014) Let $\sum_{n=0}^{\infty} A_n x^n$ be the MacLaurin series for e^{3x}. Find A_5.

(7) (Final 2013) Let $f(x) = x^2 \sin(x^3)$. Find $f^{11}(0)$.