Math 101 – WORKSHEET 30
POWER SERIES

(1) Which of the following is a power series:

$$\square \sum_{n=0}^{\infty} \frac{n!(x-3)^n}{2^{2^n}} \qquad \square \sum_{n=0}^{\infty} \frac{3}{n!}\left(e^x\right)^n$$

1. The interval of convergence

(2) Find the interval of convergence and radius of convergence of the power series

(a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-1)^n}{n}$

(b) $\sum_{n=0}^{\infty} n! x^n$

(c) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

(d) (Final, 2014, variant) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n(n^2+1)}$

(e) (Final, 2011) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{\log(n+2)}$

Date: 22/3/2017, Worksheet by Lior Silberman.

(3) Consider a power series $\sum_{n=0}^{\infty} A_n (x-5)^n$.

(a) The power series converges at $x = -3$. Show that it converges at $x = 10$.

(b) The power series diverges at $x = 15$. Show that it diverges at $x = -15$.

(c) Can you tell if the series converges at $x = 14$? What can you say about the radius of convergence?

2. Manipulating power series

(4) Let $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, $g(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$.

(a) Find the power series representation of $f'(x)$. What is $f(x)$?

(b) Find the power series representation of $g'(x)$. What is $g'(x)$? What is $g(x)$?

(c) Find the power series representation of $\int_0^x f(-t^2)\,\mathrm{d}t$.