Math 101 – WORKSHEET 16
APPROXIMATE INTEGRATION

(1) (Final 2012) Let $I = \int_1^2 \frac{1}{x}\,\mathrm{d}x$.

(a) Write down Simpson's rule approximation for I using 4 points (call it S_4)

(b) Without computing I, find an upper bound for $|I - S_4|$. You may use the fact that if $\left|f^{(4)}(x)\right| \le K$ on $[a, b]$ then the error in the approximation with n points has magnitude at most $K(b-a)^5/180n^4$.

(2) (Final 2015) Consider $I = \int_0^2 (x-3)^5\,\mathrm{d}x$.

(a) Write down the Simpson's rule approximation to I with $n = 6$. You may leave your answers in calculator-ready form.

(b) Which method of approximating I results in a smaller error bound: the Midpoint Rule with $n = 100$ intervals, or Simpson's Rule with $n = 10$ intervals? Justify your answer. You may use the formulas $|E_{\mathrm{M}}| \le \frac{K(b-a)^3}{24n^2}$ and $|E_S| \le \frac{L(b-a)^5}{180n^4}$ where K is an upper bound for $|f''(x)|$ and L is an upper bound for $\left|f^{(4)}(x)\right|$.

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(3) (Final 2008) Let $I = \int_0^1 \cos(x^2)\,\mathrm{d}x$. It can be shown that the fourth derivative of $\cos(x^2)$ has absolute value at most 60 on $[0,1]$. Find n such the Simpson's rule approximation to I using n points has error less than or equal to 0.001. You may use that that if $\left|f^{(4)}(t)\right| \leq K$ for $a \leq t \leq b$ then error in using Simpson's rule to approximate $\int_a^b f(x)\,\mathrm{d}x$ has absolute value less than or equal to $K(b-a)^5/180n^4$.

(4) Let $I = \int_4^6 \sin(\sqrt{x})\,\mathrm{d}x$. Find n such that estimating I using the midpoint rule and n points will have an error of at most $\frac{1}{3000}$. You may use that the absolute error in estimating $\int_a^b f(x)\,\mathrm{d}x$ using the midpoint rule and n points is at most $K(b-a)^3/24n^2$ whenever $\left|f^{(2)}(x)\right| \leq K$ for $a \leq x \leq b$.