8. Volumes (20/1/2017)

Goals.

(1) Area between curves (slicing horizontally)
(2) Computing Volumes
(3) Quiz

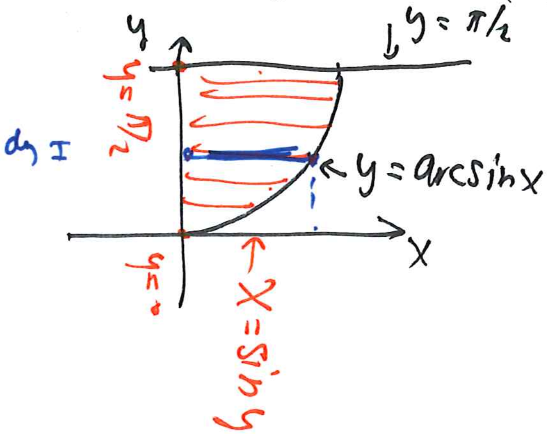
Last Time: Area between curves, slicing vertically.

Worksheet 1

Math 101 – WORKSHEET 8
AREA BETWEEN CURVES, VOLUMES

(1) Find the total area of the following planar regions. It will be useful to sketch the region first.

(a) The finite region bounded by the y-axis, the graph of $y = \arcsin(x)$ and the line $y = \frac{\pi}{2}$.

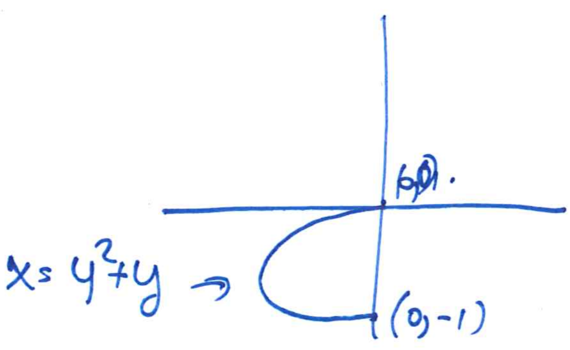


area $= \int_{y=0}^{y=\pi/2} \sin y\,dy = 1$

slice horizontally.
slice at height y has thickness dy length x, where y = arcsin x
so x = sin y so area is
$\int_{y=0}^{y=\pi/2} \sin y\,dy = [\cos y]_0^{\pi/2} = 1$

(b) (Quiz, 2015) The finite region to the left of the y-axis and to the right of the curve $x = y^2 + y$.

(imagine it said $y = x^2 + x$)

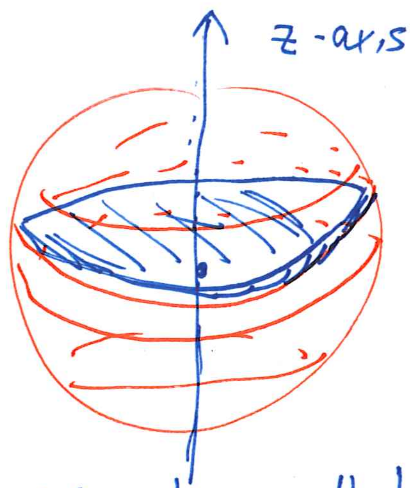


Date: 20/1/2017, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

Volumes

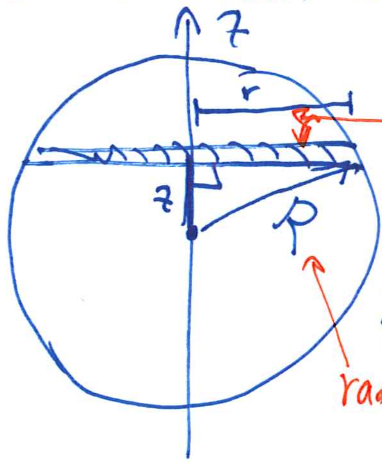
Example: Find the volume of a ball of radius ~~1~~ 1.

Method: (1) Picture
(2) Chop up
(3) Geometry on slices
(4) sum



If we slice perpendicular to z-axis, slices will have circular cross-section. The slice at height z, ~~width~~ thickness dz looks like a "cylinder" or "wafer" of radius r, thickness dz so volume of $\pi r^2 dz$. Total volume is $\int_{z=-1}^{z=1} \pi r^2 dz$ but r depends on z.

Look at ball from side:



slice is perpendicular to NS axis, so

by Pythagoras $R^2 = z^2 + r^2$

so $r^2 = R^2 - z^2$, volume is $\pi \int_{z=-R}^{z=R} (R^2 - z^2)\,dz = 2\pi \int_0^R (R^2 - z^2)\,dz =$

$R^2 - z^2$ is even

$$= 2\pi \left[R^2 z - \frac{z^3}{3} \right]_0^R = 2\pi \left[R^3 - \frac{R^3}{3} \right] = \frac{4\pi}{3} R^3$$