Math 101 – WORKSHEET 27
ALTERNATING SERIES

1. Converge or Diverge?

(1) Determine, with explanation, whether the following series converge or diverge.
   (a) (Alternating harmonic series) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

   (b) $1 - \frac{1}{4} + \frac{1}{3} - \frac{1}{16} + \frac{1}{5} - \frac{1}{36} + \frac{1}{7} - \frac{1}{64} + \frac{1}{9} - \frac{1}{100} + \frac{1}{11} - \frac{1}{144} + \cdots$

   (c) (Final 2014) $\sum_{n=1}^{\infty} \frac{n\cos(\pi n)}{2^n}$

   (d) (Final 2011) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p} = 1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \cdots$ (your answer will depend on p)

(2) Power series
   (a) (Final 2013, variant) Decide whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}(x+2)^n$ converges or diverges at $x = -1$ and at $x = -3$.

   (b) Decide whether the series $\sum_{n=1}^{\infty} nx^n$ converges or diverges at $x = 1$ and $x = -1$.

Date: 14/3/2016, Worksheet by Lior Silberman.

2. Error estimates

(3) (a) It is known that $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots = \log 2$. How many terms are needed for the error to be less than 0.01?

(b) It is known that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots = \frac{\pi}{4}$. How many terms are needed for the error to be less than 0.001?

(4) (MacLaurin expansions)

(a) It is known that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. How close is $\frac{1}{2} - \frac{1}{6} + \frac{1}{24}$ to $\frac{1}{e}$? How many terms are needed to approximate $\frac{1}{e}$ to within $\frac{1}{1000}$?

(b) The *error function* is (roughly) given by $\mathrm{erf}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(2n+1)} x^{2n+1}$. How many terms are needed to approximate $\mathrm{erf}(\frac{1}{10})$ to within 10^{-11}?