Math 101 – WORKSHEET 25
THE INTEGRAL TEST

1. Review of improper integrals

(1) Show that $\int_2^\infty \frac{\mathrm{d}x}{x}$ diverges.

(2) Show that $\int_2^\infty \frac{\mathrm{d}x}{x^3+5}$ converges.

(3) Evaluate $\int_0^\infty xe^{-x}\,\mathrm{d}x$.

2. Applying the integral test

(4) Decide if each series converges or diverges
   (a) $\sum_{n=1}^\infty \frac{1}{n^p}$ (your answer will depend on p!)

   (b) $\sum_{n=1}^\infty \frac{n}{e^n}$

   (c) (Final 2014) $\sum_{n=2}^\infty \frac{1}{n(\log n)^p}$ (your answer will depend on p!)

   (d) $\sum_{n=1}^\infty \frac{1}{n^2+1}$

(5) The integral $\int_2^\infty \frac{x+\sin x}{1+x^2}\,\mathrm{d}x$ diverges. Why can't we use the integral test to conlcude that $\sum_{n=2}^\infty \frac{n+\sin n}{1+n^2}$ diverges as well?

Date: 9/3/2016, Worksheet by Lior Silberman.

3. Tail estimates (not examinable)

(6) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(a) Show that $\sum_{n=N+1}^{\infty} \frac{1}{n^2} \le \frac{1}{N}$.

(b) How many terms to we need to include to get an answer accurate to 10^{-5}?

(7) (The harmonic series)

(a) Show that $\sum_{n=1}^{N} \frac{1}{n} \ge \log(N+1)$

(b) Show that $\sum_{n=1}^{N} \frac{1}{n} \le (1 - \log 2) + \log(N+1)$