Math 101 – WORKSHEET 22
SEQUENCES

1. Skill 1: expression for sequences

(1) For each of the following sequences, write a formula for the general term
   (a) $\{1, 2, 3, 4, 5, 6, \cdots\}$

   (b) $\left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}, \cdots\right\}$

   (c) $\{3, 7, 11, 15, 19, \cdots\}$

   (d) $\left\{\frac{7}{9}, \frac{7}{27}, \frac{7}{81}, \frac{7}{243}, \frac{7}{729}, \frac{7}{3187}, \cdots\right\}$

   (e) $\left\{\frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \frac{5}{32}, \frac{3}{32}, \frac{7}{128}, \frac{1}{32}, \frac{9}{512}, \frac{5}{512} \cdots\right\} = \left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{8}, \frac{4}{16}, \frac{5}{32}, \frac{6}{64}, \frac{7}{128}, \frac{8}{256}, \frac{9}{512}, \frac{10}{1024}, \cdots\right\}$

   (f) $\{1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, \cdots\}$

   (g) $\left\{0, \frac{3}{8}, \frac{2}{27}, \frac{5}{64}, \frac{4}{125}, \frac{7}{216}, \frac{6}{343}, \frac{9}{512}, \frac{8}{729}, \frac{11}{1000}, \cdots\right\}$

2. Skill 2: limits of sequences

(2) Determine if the sequences is convergent of divergent. If convergent, evaluate the limit.
   (a) $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$

   (b) $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$

   (c) $\{\sin(n)\}_{n=5}^{\infty}$

   (d) $\left\{\sin(\frac{1}{n})\right\}_{n=1}^{\infty}$

Date: 2/3/2016, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

(3) Further problems
  (a) Does $\lim_{n\to\infty} \frac{n}{\sqrt{n+1000}}$ exist?

  (b) $\lim_{n\to\infty} \frac{n}{2^n} =$

  (c) (Math 103 final, 2014) Consider the sequence $\{a_n\}_{n=1}^{\infty} = \left\{1, 0, \frac{1}{2}, 0, 0, \frac{1}{3}, 0, 0, 0, \frac{1}{4}, 0, 0, 0, 0, \frac{1}{5}, \cdots\right\}$. Decide whether $\lim_{n\to\infty} a_n = 0$.

3. Tool: Squeeze Theorem

(4) Determine if the sequences is convergent of divergent. If convergent, evaluate the limit.
  (a) (Final 2013) $\left\{(-1)^n \sin\left(\frac{1}{n}\right)\right\}_{n=1}^{\infty}$.

  (b) (Final 2011) $\left\{\frac{\sin(n)}{\log(n)}\right\}_{n=2}^{\infty}$ (why do we have $n \geq 2$ here?)

  (c) (Math 105 Final 2012) $a_n = 1 + \frac{n! \sin(n^3)}{(n+1)!}$.