Math 101 – WORKSHEET 20
THE CENTRE OF MASS

1. Point masses

(1) Three masses are placed at the points $(-1, 0), (1, 0), (0, 5)$. Find the centre of mass of the configuration.

    (a) When the masses are equal,

    (b) When the mass at $(-1, 0)$ is twice as large as the others.

(2) The mass of the Earth is about 6×10^{24}kg. The mass of the Moon is about 7.2×10^{22}kg. The distance between the centres of the Earth and the Moon is $3.8 \cdot 10^{5}$km. Where is the centre of mass of the Earth–Moon system? [aside: the radius of the Earth is about 6400km].

(3) A tenderizing hammer consists of a 1kg head attached to a 30cm-long shaft massing 400g.

    (a) Find the centre of mass of the hammer.

    (b) What fraction of the mass of the hammer is on each side of the centre of mass?

If the region R lies between the graphs of $f(x)$ and $g(x)$, then its area is $A = \int_a^b (f(x) - g(x)) \, dx$ and its centre of mass is at

$$X = \frac{1}{A} \int_a^b x \, (f(x) - g(x)) \, dx; \qquad Y = \frac{1}{A} \frac{1}{2} \int_a^b \left(f^2(x) - g^2(x)\right) dx \,.$$

If the density is non-uniform (say given by $\rho = \rho(x)$) then all integrals need a factor $\rho(x)$ in them.

(3) (Final 2013) The region R consists of a semicircle of radius 3 on top of a rectangle of width 6 and height 2. Find its centre of mass.
  (a) Using the formulas above

  (b) Using the known locations of the centres of mass of the semicircle and the rectangle.

(4) Find the centre of mass of the region lying below the x axis, between the branches of $\log |x|$.