Math 101 – WORKSHEET 17
APPROXIMATE INTEGRATION

1. Approximate integration

(1) Let $f(x) = \sin(x^2)$. Estimate $\int_0^1 f(x)\,\mathrm{d}x$ using the trapezoid rule, the midpoint rule, and Simpson's rule, with $n = 4$ in all cases. You may leave your answers in calculator-ready form.

(2) (Final 2009) Give the Simpson's rule approximation to $\int_0^2 \sin(e^x)\,\mathrm{d}x$ using 4 equal subintervals.

(3) (Final 2012) Let $I = \int_1^2 \frac{1}{x}\,\mathrm{d}x$.
  (a) Write down Simpson's rule approximation for I using 4 points (call it S_4)

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(b) Without computing I, find an upper bound for $|I - S_4|$. You may use the fact that if $\left|f^{(4)}(x)\right| \leq K$ on $[a, b]$ then the error in the approximation with n points has magnitude at most $K(b-a)^5/180n^4$.

(4) (Final 2008) Let $I = \int_0^1 \cos(x^2)\,\mathrm{d}x$. It can be shown that the fourth derivative of $\cos(x^2)$ has absolute value at most 60 on $[0, 1]$. Find n such the Simpson's rule approximation to I using n points has error less than or equal to 0.001. You may use that that if $\left|f^{(4)}(t)\right| \leq K$ for $a \leq t \leq b$ then error in using Simpson's rule to approximate $\int_a^b f(x)\,\mathrm{d}x$ has absolute value less than or equal to $K(b-a)^5/180n^4$.

(5) Let $I = \int_4^6 \sin(\sqrt{x})\,\mathrm{d}x$. Find n such that estimating I using the midpoint rule and n points will have an error of at most $\frac{1}{3000}$. You may use that the absolute error in estimating $\int_a^b f(x)\,\mathrm{d}x$ using the midpoint rule and n points is at most $K(b-a)^3/24n^2$ where $\left|f^{(4)}(x)\right| \leq K$ for $a \leq x \leq b$.