First Name: ______________________ Last Name: ______________________

Student-No: ______________________ Section: ______________________

Very short answer questions

1. $\boxed{\text{2 marks}}$ Each part is worth 1 mark. Please write your answers in the boxes.

(a) Evaluate $\sin\left(\dfrac{2\pi}{3}\right)$.

Answer: $\sqrt{3}/2$

Solution:

$$\begin{aligned}\sin 2\pi/3 &= \sin(\pi - 2\pi/3)\\ &= \sin(\pi/3)\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

Else draw the appropriate $2:1:\sqrt{3}$ triangle.

(b) Compute $\lim\limits_{t\to-3}\left(\dfrac{1-t}{\cos(t)}\right)$.

Answer: $4/\cos(3)$

Solution:

$$\lim_{t\to-3}\left(\frac{1-t}{\cos(t)}\right) = \frac{\lim\limits_{t\to-3}(1-t)}{\lim\limits_{t\to-3}\cos(t)} \qquad = 4/\cos(-3) = 4/\cos(3)$$

Short answer questions — you must show your work

2. $\boxed{\text{4 marks}}$ Each part is worth 2 marks.

(a) Find all numbers a such that $x = -1$ is a root of $x^3 + a^2x^2 + 3a = 0$.

Solution: Setting $x = -1$ we need to solve

$$(-1) + a^2(1) + 3a = 0\,,$$

that is

$$a^2 + 3a - 1 = 0\,.$$

By the quadratic formula the solutions are

$$a = \frac{-3 \pm \sqrt{13}}{2}\,.$$

(b) Compute the limit $\lim_{x\to 3} \dfrac{x-3}{x^2-4x+3}$

Solution: If try naively then we get 0/0, so we simplify first:

$$\frac{x-3}{x^2-4x+3} = \frac{x-3}{(x-3)(x-1)} = \frac{1}{x-1}$$

Hence the limit is $\lim_{x\to 3} \dfrac{1}{x-1} = \dfrac{1}{3-1} = \dfrac{1}{2}$.

Long answer question — you must show your work

3. $\boxed{\text{4 marks}}$ Compute the limit $\lim_{x\to 2} \dfrac{\sqrt{x+7}-\sqrt{6-x}}{2x-4}$.

Solution: The numerator has $\lim_{x\to 2}\left(\sqrt{x+7}-\sqrt{6-x}\right) = \sqrt{2+7}-\sqrt{6-2} = 1$ while the denominator tends to zero, so the limit does not exist. More precisely, the function blows up with the numerator positive (close to 1) while the denominator is positive for $x > 2$ and negative for $x < 2$. We conclude that

$$\lim_{x\to 2^-} \frac{\sqrt{x+7}-\sqrt{6-x}}{2x-4} = -\infty$$

and

$$\lim_{x\to 2^+} \frac{\sqrt{x+7}-\sqrt{6-x}}{2x-4} = +\infty\,.$$