

MATH 100 – WORKSHEET 22
L'HÔPITAL'S RULE

1. STATEMENT

Theorem. Let f, g be diff. near $x = a$. Suppose $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ while $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} = L$. Then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ exists and equals L .

This also works if $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ exists in the extended sense ($L = +\infty$ or $L = -\infty$), if $\lim_{x \rightarrow a} f(x), \lim_{x \rightarrow a} g(x)$ are both infinite in the extended sense rather than zero, or if we take $\lim_{x \rightarrow \infty} (a = \infty)$

(1) Evaluate $\lim_{x \rightarrow 1} \frac{\ln x}{x-1}$.

(2) Evaluate $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}$.

(3) Do (2) using a 2nd-order Taylor expansion.

(4) Given that $f(2) = 5, g(2) = 3, f'(2) = 7$ and $g'(2) = 4$ find $\lim_{x \rightarrow 3} \frac{f(2x-4) - g(x-1)}{g(x^2-7)}$.

(5) Evaluate $\lim_{x \rightarrow 0^+} \frac{e^x}{x}$.

(6) Evaluate $\lim_{x \rightarrow \infty} x^2 e^{-x}$.

(7) Evaluate $\lim_{x \rightarrow 0^+} x \ln x$.

(8) Evaluate $\lim_{x \rightarrow \infty} x^n e^{-x}$.

(9) Suppose $a > 0$. Evaluate $\lim_{x \rightarrow \infty} x^{-a} \ln x$.

(10) Evaluate $\lim_{x \rightarrow 0} (2x + 1)^{1/\sin x}$.