MATH 100 – WORKSHEET 15
ESTIMATES ON TAYLOR APPROXIMATIONS

1. Taylor approximations

The nth order Taylor expansion of $f(x)$ about $x = a$ is the polynomial

$$T_n(x) = f(a) + f'(a)(x-a) + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

(1) Find the 1st and 2nd order Taylor expansions of $x^{3/2}$ about $x = 4$ and use them to approximate $(4.1)^{3/2}$.

(2) Find the 2nd order Taylor expansion of $x^{3/2} + 3x$ about $x = 4$.

(3) Find the 8th order expansion of $f(x) = e^{x^2} + \cos(5x)$. What is $f^{(6)}(0)$?

2. Error estimates

Let $R_1(x) = f(x) - T_1(x)$ be the *remainder*. Then there is c between a and x such that

$$R_1(x) = \frac{f^{(2)}(c)}{2!}(x-a)^2$$

(1) Estimate the error in the linear approximation to $(4.1)^{3/2}$.

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Let $R_n(x) = f(x) - T_n(x)$ be the *remainder*. Then there is c between a and x such that

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

(2) Estimate the error in the quadratic approximation to $(4.1)^{3/2}$.

(3) Estimate the error in the 4th order approximation to $\cos(0.5)$