MATH 100 – WORKSHEET 14
TAYLOR POLYNOMIALS

1. Taylor expansion of e^x

(1) Let $f(x) = e^x$
   (a) Find $f(0), f'(0), f^{(2)}(0), \cdots$
   (b) Find a simple polynomial $T_0(x)$ such that $T_0(0) = f(0)$.
   (c) Find a simple polynomial $T_1(x)$ such that $T_1(0) = f(0)$ and $T_1'(0) = f'(0)$.
   (d) Find a simple polynomial $T_2(x)$ such that $T_2(0) = f(0)$, $T_2'(0) = f'(0)$ and $T_2^{(2)}(0) = f^{(2)}(0)$.
   (e) Find a simple polynomial $T_3(x)$ such that $T_3^{(k)}(0) = f^{(k)}(0)$ for $0 \le k \le 3$.

2. Do the same with $f(x) = \ln x$ about $x = 1$.

2. General formula

The nth order Taylor expansion of $f(x)$ about $x = a$ is the polynomial
$$T_n(x) = c_0 + c_1(x - a) + \cdots + c_n(x - a)^n$$
where $c_k = \frac{f^{(k)}(a)}{k!}$.

(1) Find the 4th order Maclaurin expansion of $\frac{1}{1-x}$.

(2) Find the nth order expansion of $\cos x$.

3. New from old

(1) Find the 3rd order Taylor expansion of $\sqrt{x}$ about $x = 4$ and use it to approximate $\sqrt{4.1}$.

(2) Find the 3rd order Taylor expansion of $\sqrt{x} + 3x$ about $x = 4$.

(3) Find the 8th order expansion of $f(x) = e^{x^2} + \cos(2x)$. What is $f^{(6)}(0)$?