

**MATH 100 – WORKSHEET 10**  
**LOGARITHMS AND THEIR DERIVATIVES**

1. INVERSE TRIG & DIFFERENTIATION

<b>Fact.</b> $\frac{d \arcsin x}{dx} = \frac{1}{\sqrt{1-x^2}}, \frac{d \arctan x}{dx} = \frac{1}{1+x^2}.$
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(1) The angle  $\theta$  lies in the range  $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$  and satisfies  $\sin(\theta) = 0.4$ . find  $\tan \theta$ .

(2) (Final 2011) Find the derivative of  $\arcsin(3x + 1)$

2. REVIEW OF LOGARITHMS

$\log_b(b^x) = b^{\log_b x} = x$
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$\log_b(xy) = \log_b x + \log_b y$
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$\log_b(x^y) = y \log_b x$
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$\log_b \frac{1}{x} = -\log_b x$
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(1)  $\log(e^{10}) =$

$\log(2^{100}) =$

(in terms of  $\log 2$ )

(2) A variant on *Moore's Law* states that computing power doubles every 18 months.

Suppose computers today can do  $N_0$  operations per second.

(a) Write a formula for the power of computers  $t$  years into the future:

- Computers  $t$  years from now will be able to do  $N(t)$  operations per second where

$$N(t) =$$

(b) A computing task would take 10 years for today's computers. Suppose we wait 3 years and then start the computation. When will we have the answer?

(c) At what time will computers be powerful enough to complete the task in 6 months?

### 3. DIFFERENTIATION

$$\boxed{(\log x)' = \frac{1}{x}}$$

$$\boxed{f' = f \times (\log f)'}$$

(1) Differentiate

(a)  $\frac{d(\log(ax))}{dx} =$

$$\frac{d}{dt} \log(t^2 + 3t) =$$

(b)  $\frac{d}{dx} x^2 \log(1 + x^2) =$

$$\frac{d}{dr} \frac{1}{\log(2 + \sin r)} =$$

(c) Find  $y'$  if  $\log(x + y) = e^y$