MATH 100 – WORKSHEET 4
CONTINUITY, HORIZONTAL ASYMPTOTES, THE DERIVATIVE

1. The Intermediate Value Theorem

(1) Show that:
   (a) $f(x) = 2x^3 - 5x + 1$ has a zero in $0 \leq x \leq 1$.

   (b) There is $x > 0$ for which $\frac{1}{x} = \sin x$.

(2) (Final 2011) Let $y = f(x)$ be continuous with domain $[0, 1]$ and range in $[3, 5]$. Show the line $y = 2x + 3$ intersects the graph of $y = f(x)$ at least once.

2. Horizontal Asymptotes

(1) Evaluate the following limits:
   (a) $\lim_{x\to\infty} \frac{x^2+1}{x-3}$

   (b) $\lim_{x\to\infty} \frac{x^2+8}{2x^3-1}$

   (c) $\lim_{x\to\infty} \frac{\sqrt{x^4+\sin x}}{x^2-\cos x}$

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(d) $\lim_{x\to-\infty}\left(\sqrt{x^2+2x}-\sqrt{x^2-1}\right)$

(2) Find the horizontal and vertical asymptotes of $\frac{x^2+x+1}{x^2-4}$

3. Calculate the derivatives

Definition. $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

(1) Find $f'(a)$ if
   (a) $f(x)=x^2$, $a=3$.
   (b) $f(x)=\frac{1}{x}$, any a.
   (c) $f(x)=x^3-2x$, any a. (you may use $(a+h)^3=a^3+3a^2h+3ah^2+h^3$).