MATH 100 – WORKSHEET 3
CONTINUITY

1. Continuity

(1) Which of these functions are continuous? Why?

(a) $f(x) = \begin{cases} x & x < 0 \\ \cos x & x \geq 0 \end{cases}$

(b) $f(x) = \begin{cases} x & x < 0 \\ \sin x & x \geq 0 \end{cases}$

(2) Let $f(x) = \frac{x^3 - x^2}{x-1}$.

(a) Why is $f(x)$ discontinuous at $x = 1$?

(b) Find b such that $g(x) = \begin{cases} f(x) & x \neq 1 \\ b & x = 1 \end{cases}$ is continouous everywhere.

(3) Find c, d such that $f(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ c & x = 1 \\ d - x^2 & x > 1 \end{cases}$ is continuous.

(4) Where are the following functions continuous?

(a) $\frac{x^2+2x+1}{2+\cos x}$

(b) $\frac{2+\cos x}{x^2+2x+1}$

(c) $\log\left((\sin x)^2\right)$

Date: 17/9/2015, Worksheet by Lior Silberman.

(Final 2011) Suppose f, g are continuous such that $g(3) = 2$ and $\lim_{x\to 3}(xf(x) + g(x)) = 1$. Find $f(3)$.

2. The Intermediate Value Theorem

Theorem. *Let $f(x)$ be continuous for $a \le x \le b$. Then $f(x)$ takes every value between $f(a), f(b)$.*

(1) Show that:
   (a) $f(x) = 2x^3 - 5x + 1$ has a zero in $0 \le x \le 1$.

   (b) $\cos x = x$ has a solution.

(2) (Final 2011) Let $y = f(x)$ be continuous with domain $[0, 1]$ and range in $[3, 5]$. Show the line $y = 2x + 3$ intersects the graph of $y = f(x)$ at least once.