

**MATH 100 – WORKSHEET 11**  
**EXPONENTIAL GROWTH AND DECAY**

1. EXPONENTIALS

Growth/decay described by the *differential equation*

$$\frac{dy}{dt} = ky,$$

**Solution:**  $y =$

- (1) A pair of invasive Opossums arrives in BC in 1930. Unchecked, Opossums can triple their population annually. Assume that this in fact happens.
- (a) At what time will there be 1000 Opossums in BC? 10,000 Opossums?

(b) Write a differential equation expressing the growth of the Opossum population with time.

- (2) A radioactive sample decays according to the law

$$\frac{dm}{dt} = -km.$$

- (a) Suppose that one-quarter of the sample remains after 10 hours. What is the half-life?
- (b) A 100-gram sample is left unattended for three days. How much of it remains?

- (3) Euler found that the tension in a wire wound around a cylinder increases according to the equation

$$\frac{dT}{d\alpha} = \mu T$$

where  $\mu$  is the coefficient of friction and  $\alpha$  is the angle around the cylinder.

- (a) When mooring a large ship a cable is wound around a bollard. It is found that when looping the cable once around the bollard, the ratio of tensions at the two ends of the rope is 20. What is the coefficient of friction?
- (b) The rope is wound 3.5 times around the bollard. What is the force gain?

## 2. NEWTON'S LAW OF COOLING

**Fact 1.** When a body of temperature  $T$  is placed in an environment of temperature  $T_{\text{env}}$ , the rate of change of  $T$  is negatively proportional to the temperature difference  $T - T_0$ . In other words, there is  $k$  such that

$$T' = -k(T - T_{\text{env}}).$$

- *key idea:* change variables to the temperature difference. Let  $y = T - T_{\text{env}}$ . Then

$$\frac{dy}{dt} = \frac{dT}{dt} - 0 = -ky$$

so there is  $C$  for which

$$y(t) = Ce^{-kt}.$$

Solving for  $T$  we get:

$$T(t) = T_{\text{env}} + Ce^{-kt}.$$

Setting  $t = 0$  we find  $T(0) = T_{\text{env}} + C$  so  $C = T(0) - T_{\text{env}}$  and

$$T(t) = T_{\text{env}} + (T(0) - T_{\text{env}})e^{-kt}.$$

**Corollary 2.**  $\lim_{t \rightarrow \infty} y(t) = T_0$ .

**Example** (Final Exam, 2010). When an apple is taken from a refrigerator, its temperature is  $3^\circ\text{C}$ . After 30 minutes in a  $19^\circ\text{C}$  room its temperature is  $11^\circ\text{C}$ .

- (1) Write the *differential equation* satisfied by the temperature  $T(t)$  of the apple.
- (2) Find the temperature of the apple 90 minutes after it is taken from the refrigerator, expressed as an integer number of degrees Celsius.
- (3) Determine the time when the temperature of the apple is  $16^\circ\text{C}$ .