

MATH 100 – WORKSHEET 2

LIMITS

(1) Let $f(x) = \frac{x-3}{x^2-x-6}$. What is $\lim_{x \rightarrow 3} f(x)$?

(2) What about $\lim_{x \rightarrow 2} f(x)$? What about $\lim_{x \rightarrow 2^+} f(x)$, $\lim_{x \rightarrow 2^-} f(x)$?

(3) Evaluate

(a) $\lim_{x \rightarrow 1} \sin(\pi x)$

(b) $\lim_{x \rightarrow \pi^+} \frac{1}{\sin(x)}$, $\lim_{x \rightarrow \pi^-} \frac{1}{\sin(x)}$.

(4) Either evaluate the limit or explain why it does not exist

(a) $\lim_{x \rightarrow 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ 1 & x = 1 \\ 2 - x^2 & x > 1 \end{cases}$.

(b) $\lim_{x \rightarrow 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ 1 & x = 1 \\ 4 - x^2 & x > 1 \end{cases}$.

2. LIMIT LAWS

Fact. *Limits respect arithmetic operations and standard functions (e^x , \sin , \cos , \log , ...) as long as everything is well-defined.*

(beware especially of division by zero)

(1) Evaluate using the limit laws:

(a) $\lim_{x \rightarrow 2} \frac{x+1}{4x^2-1}$

(b) $\lim_{x \rightarrow 1} \frac{e^x(x-1)}{x^2+x-2}$.

(2) Evaluate:

(a) $\lim_{x \rightarrow 0} \frac{\sqrt{4+x}-2}{x}$.

(b) $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{\pi}{x}\right)$.

(c) Suppose that $1 \leq f(x) \leq (x^2 - 2x + 2)$ for all x . Find $\lim_{x \rightarrow 1} f(x)$.