**HOMEWORK 2: MATH 265, L Keshet** (**Final version**) Due in class on September 29, 2010

NOTE: Most problems on this assignment are straightforward. Problem 4 may take a bit more time and effort.

Problem 1: In each case, solve the following second order ODEs for $y(t)$:

(a) $y'' + 2y' - 3y = 0$, and $y(0) = 1, y'(0) = 2$

(b) $y'' - 9y' + 20y = 0$ and $y(0) = 1, y'(0) = 0$

(c) $y'' - 2y' + 5y = 0$ and $y(0) = 1, y'(0) = 1$.

(d) $y'' - 2y = 0$ and $y(0) = 0, y'(0) = 2$.

Problem 2: Consider the differential equation $ay'' + by' + cy = 0$. Suppose that the two functions $y = f_1(t)$ and $y = \frac{1}{2}[f_2(t) + f_1(t)]$ are both solutions to this equation. Show that the function $f_2(t)$ is also a solution.

Problem 3: Find a value of the constant $r$ such that both $e^{rt}$ and $te^{rt}$ are solutions to the ODE

$$ay'' + by' + cy = 0$$

Problem 4: A patient is in the hospital on intravenous medication. We will denote by $I(t)$ the rate at which medication is infused (injected into the patient). Assume this is already corrected for weight of patient and that it is immediately well-mixed in the bloodstream. Let $c(t)$ denote the drug concentration (mg/L) in the bloodstream at time $t$. The drug is broken down by the liver at a constant rate $r \geq 0$ (per hr). Assume that the ODE and initial condition that describes this situation is

$$\frac{dc}{dt} = I(t) - rc, \quad c(0) = 0.$$

(a) Suppose that $I(t)$ is switched on at time 0, is constant for an hour ($I(t) = \bar{I}$ for $0 \leq t \leq 1$hr) and then switched off. Find the value of $c(t)$ during and after this period of time. (Your answer will be in terms of constants in the problem.

(b) Sketch the solution you got in part (a). (The sketch should be approximate but should be labeled carefully.)

(c) Now suppose that for a second patient, the infusion rate is periodic and the decay rate is $r = 1$ per hour, so that the ODE is

$$\frac{dc}{dt} = 1 + \sin(\pi t/6) - c, \quad c(0) = 0.$$

Solve the ODE for $c(t)$ sketch the solution.

Problem 5: A student solves a certain linear homogeneous differential equation of second order (e.g. $y'' + p(t)y' + q(t)y = 0$) and finds two solutions: $y_1(t) = 2e^t$ and $y_2(t) = e^{t-1}$. Now he would like to find the constants $c_1$ and $c_2$ such that the solution $y(t) = c_1 y_1(t) + c_2 y_2(t)$ also satisfies the initial conditions $y(0) = 1, y'(0) = 1$. The student encounters some difficulty. What is the difficulty, and why does it occur? (Trace the steps that the student might be making and help figure out why he/she runs into problems).

Problem 6: Unlike linear ODEs, for which we have results guaranteeing the existence and "good behaviour" of solutions, **nonlinear** ODEs can have all kinds of problems. Consider the simple (nonlinear) ODE

$$\frac{dy}{dt} = y^2, \quad y(0) = y_0.$$

Solve this ODE using separation of variables. Show that the solution can "blow up" (become undefined) at some finite time. For what value of $y_0$ will the solution "blow up" when $t = 2$?