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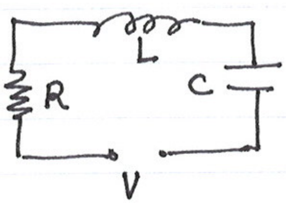
Math 265

www.math.ubc.ca/~keshet/math265

Leah Keshet
Math Annex 1111
keshet@math.ubc.ca



Kirchoff's Laws:

voltage V = sum of other voltage drops across elements.

voltage drop across	resistor :	$R\,i(t)$
" " "	capacitor :	$\dfrac{q(t)}{C}$
" " "	inductor :	$L\dfrac{di}{dt}$

<u>units</u>

$q(t)$	coulombs
$v(t)$	volts
$i(t)$	amps
L	henrys
R	ohms
C	farads

Kirchoff's Law $\Rightarrow$

$$V = L\frac{di}{dt} + R\,i(t) + \frac{q(t)}{C}$$

Also: $i(t) = \dfrac{dq}{dt}$ sub into

$$\Rightarrow \boxed{V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}}$$

Interested: in characterizing behav. of $q(t)$ given some "starting value"

$q(0) = q_0$ (given)
$i(0) = i_0$ (given)

Q: what happens as t increases?

$L, R, C \geq 0$ constants

$V(t)$ might be time-dependent (known)

<u>Second order</u>, ordinary diff'l eqn (ODE), linear

wanted: $y(t)$

Generically:

ODE: $$F\left(\frac{d^{n} y}{dt^{n}}, \frac{d^{n-1}y}{dt^{n-1}}, \ldots \frac{dy}{dt}, y, t\right) = 0$$

nth order ODE.

Initial conditions: n starting values
$$y(0) = y_0$$
$$y'(0) = y'_0$$
$$\vdots$$

⟹ <u>Initial value problem</u>

$\Rightarrow$ unique solution exists ?

? Math ?.? theorem.

In this course:

- Firstorder ODEs
- 2nd " "

Examples of First order ODEs:

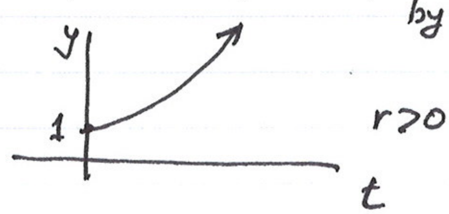
$$\frac{dy}{dt} = f(y,t) \qquad y(0) = y_0$$

<u>Example:</u> 1

$$\boxed{\frac{dy}{dt} = ry} \qquad y(0) = 1 \qquad r \text{ constant}$$

$$y(t) = C_1 e^{rt} \qquad y(0) = C_1 e^{r \cdot 0} = C_1 e^0 = C_1 = 1 \text{ by I.C.}$$

$$\Rightarrow \boxed{y(t) = e^{rt}}$$



Example ② :

non autonomous: t appears in eqn.

$$* \quad \frac{dy}{dt} = (2-t)y \qquad y(0) = 1$$

separation:
$$\frac{dy}{y} = (2-t)\,dt$$

integrate from
$$\int_{y_0}^{y(T)} \frac{dy}{y} = \int_0^T (2-t)\,dt$$

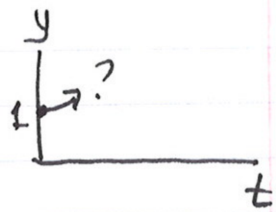
T is some "final time"

$$\ln(y) - \ln(y_0) = 2t - \frac{t^2}{2}\Big|_0^T = 2T - \frac{T^2}{2}$$

$$\ln\left(\frac{y}{y_0}\right) = 2T - \frac{T^2}{2}$$

$$\frac{y(T)}{y_0} = e^{\left(2T - \frac{T^2}{2}\right)}$$

soln
$$\boxed{y(t) = y_0\, e^{\left(2t - \frac{t^2}{2}\right)}}$$

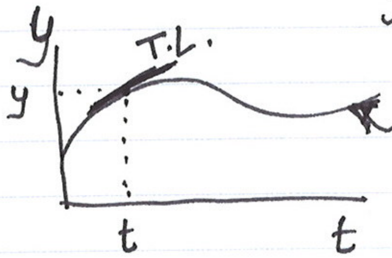


Q: so how does this fn behave?

Another approach : More qualitative :
i.e. picture

Direction Fields

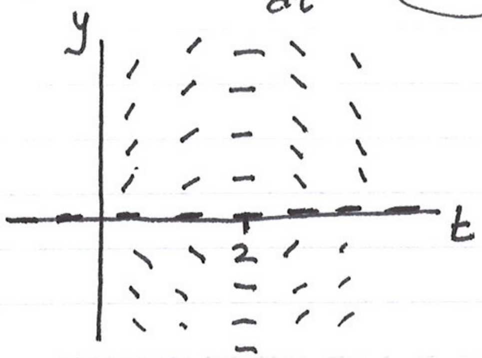
Basic idea: ODE. (such as *) gives a connection between y, t, and the slope of tangent line to y(t)



$$\frac{dy}{dt} = f(y,t)$$
slope of T.L.

T.L = tang. line

Example 2' : $\frac{dy}{dt} = (2-t)\,y$



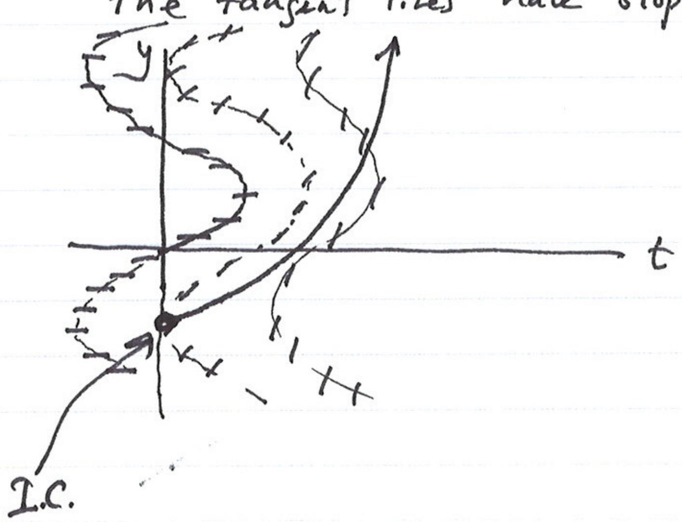
y	t	slope of T.L.	config of T.L
0		0	—
any	2	0	—
any $y>0$	<2	+	/
	>2	−	\

Example 3 : $\frac{dy}{dt} = t - \sin y \qquad y(0) = -1$

Assemble a dir. field sdn.

observe: for any curve in yt plane such that $t - \sin y = M = \text{constant}$

the tangent lines have slope M



e.g. $t - \sin y = 0$ ← slope $= 0$

$t - \sin y = 1$ ← slope $= 1$

"$(t = \sin y + 1)$"

Note: solution curves are always parallel (and never cross) those tangent vectors