Oct 6, 2010
(corrected version)

problem 23 p183

$$y'' - 4y' + 4y = \underbrace{2t^2 + 4te^{2t} + t\sin(t)}_{f(t)}$$

$y'' - 4y' + 4y = 0$ ← Hom. eqn.

$r^2 - 4r + 4 = 0$ $\quad (r-2)^2 = 0$ $\quad r = 2$ repeated

fundam sols $\quad y_1(t) = e^{2t}, \quad y_2 = te^{2t}$

UNDET. COEF:

Guess a form of partic. soln for $\quad y'' - 4y' + 4y = f(t)$ ← Nonhom eqn.

Consider Each term in $f(t)$: $\quad 2t^2 \rightarrow$ requires $A_0 + A_1 t + A_2 t^2$

$t\sin t \rightarrow$ requires $(B_0 + B_1 t)\sin t + (B_2 + B_3 t)\cos t$

$4te^{2t}$ ← this term is just a const. multiple of y_2 and will not work.

(see example 5 p179)

Need to increase by a factor of t or possibly t^2

$$4te^{2t} \rightarrow t^n(C_2 t^2 + C_1 t + C_0)e^{2t}$$

So $\quad Y_p(t) = t^n(C_1 t + C_0)e^{2t} + (B_0 + B_1 t)\sin t + (B_2 + B_3 t)\cos t + A_0 + A_1 t + A_2 t^2$

we do not want any of these terms to be a soln to hom eqn so make $n = 2$

Can find these coeffs by solving each of the three problems for Y_p

① $\quad y'' - 4y' + 4y = 2t^2 \quad \rightarrow Y_{p1}$

② $\quad y'' - 4y' + 4y = 4te^{2t} \quad \rightarrow Y_{p2}$

③ $\quad y'' - 4y' + 4y = t\sin(t) \quad \rightarrow Y_{p3}$

then ~~get~~ add up parts: $\quad Y_p(t) = Y_{p1} + Y_{p2} + Y_{p3}$

See how to solve ② on next page.

Finding Y_{p2}: We have guessed that

$$Y_{p2} = t^2(c_1 t + c_0)e^{2t} = (c_1 t^3 + c_0 t^2)e^{2t}$$

Find derivs of Y_{p2} and plug into nonhom ODE

$$Y'_{p2} = (3c_1 t^2 + 2c_0 t)e^{2t} + 2(c_1 t^3 + c_0 t^2)e^{2t}$$
$$= e^{2t}\left(2c_1 t^3 + (3c_1 + 2c_0)t^2 + 2c_0 t\right)$$

$$Y''_{p2} = \left(6c_1 t^2 + 2(3c_1 + 2c_0)t + 2c_0\right)e^{2t} + 2e^{2t}\left(2c_1 t^3 + (3c_1 + 2c_0)t^2 + 2c_0 t\right)$$
$$= e^{2t}\left(4c_1 t^3 + 2t^2(3c_1 + 2c_0 + 3c_1) + 2t(3c_1 + 2c_0 + 2c_0) + 2c_0\right)$$
$$= e^{2t}\left(4c_1 t^3 + 2t^2(6c_1 + 2c_0) + 2t(3c_1 + 4c_0) + 2c_0\right)$$

plug into Nonhom eq.

$$y'' - 4y' + 4y = 4te^{2t}$$

terms with t^3: $4c_1 - 4\cdot(2c_1) + 4(c_1) = 0 \rightarrow 0 = 0$ (no information gained here)

terms with t^2: $\cancel{2}(6c_1 + 2c_0) - \cancel{4}^{2}(3c_1 + 2c_0) + \cancel{4}^{2}(c_0) = 0$ ← (cancel factor of 2)

$$(\cancel{6c_1} + \cancel{2c_0}) - \cancel{6c_1} - \cancel{4c_0} + \cancel{2c_0} = 0$$
$$0 = 0 \quad \leftarrow \text{no information}$$

terms with t: $2(3c_1 + 4c_0) - 4(2c_0) = 4$ ← because of $4te^{2t}$ in forcing term

$$6c_1 = 4 \qquad c_1 = 2/3$$

constant terms: $2c_0 - 4(0) + 4(0) = 0 \qquad c_0 = 0$

Conclusion: $Y_{p2}(t) = \frac{2}{3}t^3 e^{2t}$