Math 265 : other examples for Sept 8, 2010

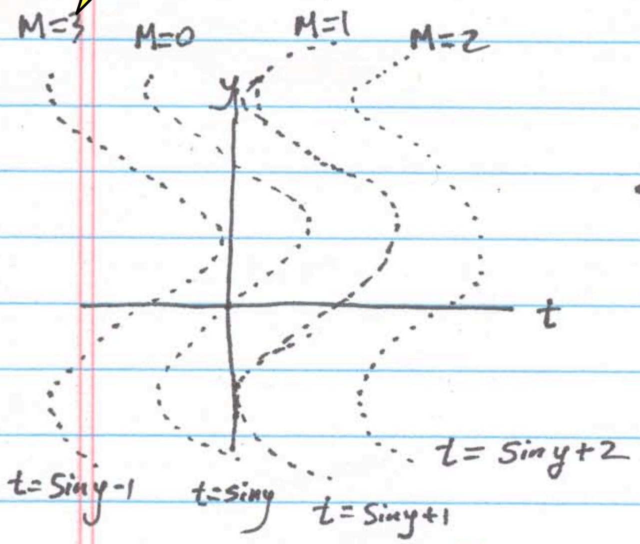
<u>Example 3</u> Direction field for the ODE

$$\frac{dy}{dt} = \underbrace{t - \sin y}_{} \qquad y(0) = -1$$

This quantity is the slope of a tangent line to solution curves (since it has to match the first derivative at any point $y(t)$).
To sketch this direction field, it is helpful to first sketch out (t,y)-curves for which

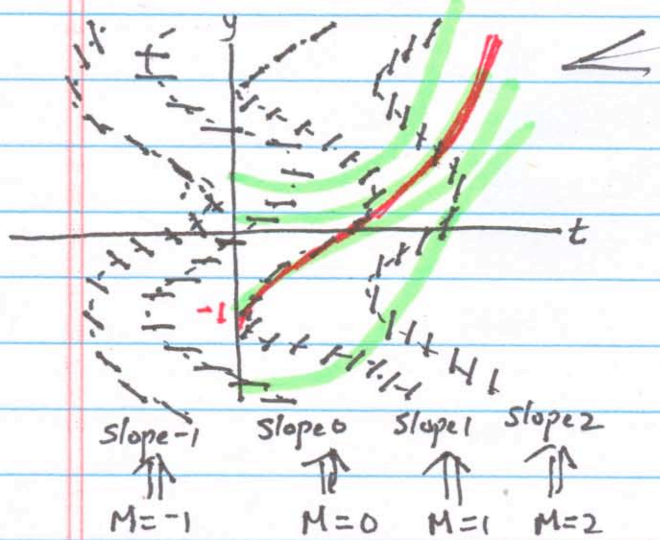
$$t - \sin y = M = \text{constant}$$

↑ then this constant will tell us what are slopes of tangents to $y(t)$ all along that locus.



← Here are a few such curves (they are all sine curves shifted along the t axis)

Next, let us draw in the tangent lines having slope M on each of these loci. This produces a direction field



Finally, let us sketch in a few solution curves $y(t)$, i.e. curves that start at some initial condition and are always parallel to this "flow"

On the above, one curve satisfies the initial condition $y(0) = -1$, shown in red

Note: Of course, we could just create a table of values for

t	y	$\frac{dy}{dt}$

as in Example 2' in the lecture, but that gets quite tedious! The above uses our graphical skills more efficiently.

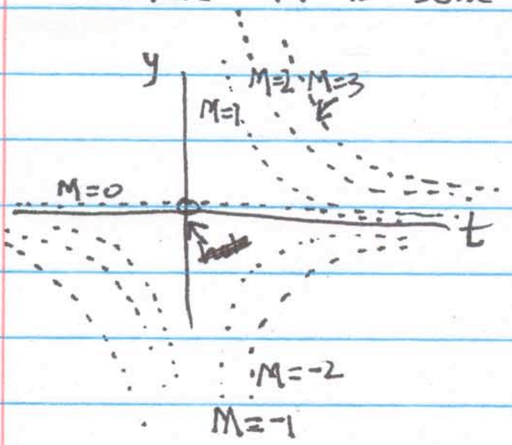
Example 4 : Draw a direction field for

$$\frac{dy}{dt} = \frac{1}{2} y t \qquad y(0) = 2$$

This says that slope of tangent lines to solutions are

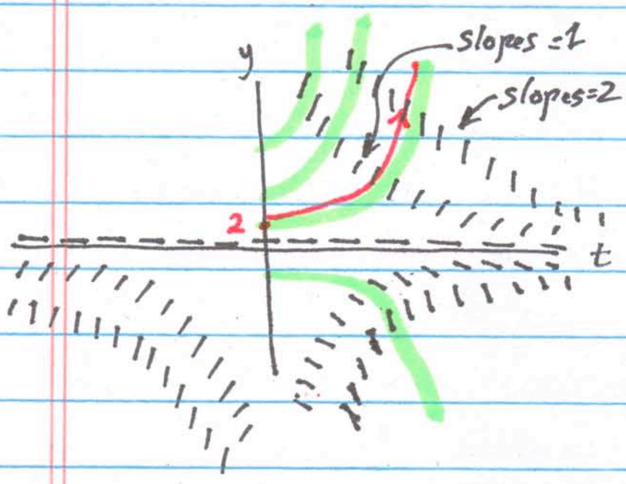
$$M = \frac{1}{2} y t \qquad \text{i.e.} \qquad y = \frac{2M}{t}$$

Let us sketch a few loci for which this relationship holds, where M is some constant



Now add tangent vectors of slope M to each curve (where $M = 1, 2, 3, -1, -2$ etc)

We get the picture shown below



Finally, add solution curve(s)

The curve we want satisfies $y(0) = 2$ and is shown in red

See also Boyce + Diprima (9th ed.) Sec 1.1