Workshop 5: Exponential and logarithmic functions, implicit differentiation, and inverse trig functions

Working in a group of 2–3 people, complete at least 2 of the following problems. After 50 minutes have elapsed, submit your solutions to your workshop instructors. You should be able to do the other problems—use them for practice!

Your group’s work will be graded on correctness, but more of your grade will depend on communication. Consider workshops as practice for your written assignments. Writing legibly (including well-labelled graphs), using complete sentences, and fully explaining your thoughts in a logical order will earn high marks.

1. Find all points \((x, y)\) on the graph of \(x^2 + y^3 = 8\) where the tangent lines have slope \(-1\). Are both \(\frac{dx}{dy}\) and \(\frac{dy}{dx}\) defined for all \((x, y)\) on the graph?

2. Find the derivative of \(y = x^x\). Then find the derivative of \(y = x^{x^x}\).

3. Consider the following log-log plot.

\[
\begin{array}{c|c|c|c|c|c}
\hline
\log x & 1 & 2 & 3 & 4 & 5 \\
\log y & 1 & 2 & 3 & 4 & 5 \\
\hline
\end{array}
\]

(a) Let \(Y = \log y\) and \(X = \log x\). Find constants \(A\) and \(B\) so that \(Y = AX + B\).

(b) Determine constants \(a\) and \(b\) such that \(y = ax^b\).

4. Find the equation of the line tangent to the graph of \(y = \log(x^2 + 4) - x\arctan\left(\frac{x}{2}\right)\) at \(x = 2\).

After you finish these problems, please submit your work to your workshop instructors. In the remaining 30 minutes of workshop, make the most of the available time and help. Work on your homework (written or online), work on practice problems, or ask your workshop instructors for help.