Quiz 8 Solutions

1. \( y = a + 10b^2x^3 - 3x^5 \), \( a, b > 0 \). Maximize tangent line slopes.

Slopes are \( y' = 30b^2x^2 - 15x^4 \).

So we set \( y'' = 0 \) and look for "critical points:"

\[
0 = 60b^2x - 60x^3 \\
0 = 60x(b^2 - x^2).
\]

\( x = 0, x = \pm b \) are the critical points.

Note that \( y''' = 60b^2 - 180x^2 \) and

\( y''(0) > 0 \) but \( y''(b) < 0 \)

So \( x = \pm b \) are local maxima and \( y'''(0) \)

is a local minimum.

\( x = \pm b \) are in fact the global maxima.

Since

\[
y'(\pm b) = 30b^2(\pm b)^2 - 15b^4
= 30b^4 - 15b^4 = 15b^4.
\]

2. \( \lim_{x \to 1} \frac{\log(x)}{2\sin(2\pi x)} = \lim_{x \to 1} \frac{y_x}{2} = \frac{1}{2} \frac{1}{2} \frac{1}{4\pi \cos(2\pi x)} \frac{1}{4\pi}
\]

Since \( \frac{y}{y} \to 0 \)