Announcements

- Related Rates (10 questions) webwork Oct 31
- Midterm info:
  
  www.math.ubc.ca/~zmurchok/180.html
  LSK 200, 6 pm, Wednesday, Oct 25
- Office Hours cancelled: Friday, Oct 20, 27
- Bonus Office Hours: Tuesday, Oct 24 at 5:00–6:30 in LSK 462

Today...

1. Curve sketching using calculus
Domain, asymptotes, and intercepts

\[ f(x) = \frac{x + 1}{(x + 3)(x - 2)} \]

Q1. What is the domain of \( f(x) \)?

A. all \( x \)
B. no \( x \)
C. \( x \neq -3, 2 \)
D. \( x \neq 3, -2 \)
Domain, asymptotes, and intercepts

\[ f(x) = \frac{x + 1}{(x + 3)(x - 2)} \]

Q2. What is the \( x \) intercept of \( f(x) \)?

A. \( x = 1, -3, 2 \)
B. \( x = -1, 3, -2 \)
C. \( x = 1 \)
D. \( x = -1 \)
Domain, asymptotes, and intercepts

\[ f(x) = \frac{x + 1}{(x + 3)(x - 2)} \]

Q3. What is \( \lim_{x \to -3^+} f(x) \) and \( \lim_{x \to -3^-} f(x) \)?

A. \( \infty, -\infty \)

B. \( -\infty, \infty \)

C. \( \frac{4}{4\cdot1} \)

D. \( \frac{2}{6\cdot-5} \)
Domain, asymptotes, and intercepts

\[ f(x) = \frac{x + 1}{(x + 3)(x - 2)} \]

Q4. What is \( \lim_{x \to \infty} f(x) \) and \( \lim_{x \to -\infty} f(x) \)?

A. \( \infty, -\infty \)

B. \( -\infty, \infty \)

C. 0, 0

D. 1, 1
Domain, asymptotes, and intercepts

\[ f(x) = \frac{x + 1}{(x + 3)(x - 2)} \]

Putting the asymptotes and intercepts on a graph:
Domain, asymptotes, and intercepts

\[ f(x) = \frac{x + 1}{(x + 3)(x - 2)} \]

- Connecting the dots...
Recall: Lennard-Jones Potential

The Lennard-Jones Potential approximates the interaction between atoms or molecules:

\[ V(r) = \varepsilon \left( \left( \frac{R}{r} \right)^{12} - 2 \left( \frac{R}{r} \right)^{6} \right) \]

- Classes and assignments... we found...
Recall: Lennard-Jones Potential

\[ V(r) \]

\[ \lim_{r \to 0^+} V(r) = \infty \]

\[ \lim_{r \to \infty} V(r) = 0 \]

\[ V'(R) = 0 \]
Recall: Lennard-Jones Potential

\[ V(r) \]

\[ \lim_{r \to 0^+} V(r) = \infty \]

\[ r \text{ intercept} \]

\[ \lim_{r \to \infty} V(r) = 0 \]

\[ V'(R) = 0 \]

Q5. Where is \( V(r) \) increasing?

A. \( 0 < r < R \)
B. \( R < r \)
C. for all \( r \)
D. \( r \neq R \)
Recall: Lennard-Jones Potential

\[ V(r) \]

\[ \lim_{r \to 0^+} V(r) = \infty \]

\[ \lim_{r \to \infty} V(r) = 0 \]

\[ V'(R) = 0 \]

Q6. Where is \( V(r) \) decreasing?

A. \( 0 < r < R \)

B. \( R < r \)

C. for all \( r \)

D. \( r \neq R \)
Increasing and decreasing functions

- Suppose $f(x)$ is defined on an interval $I$.
- $f$ is increasing on $I$ if for every $a < b$ in $I$, $f(a) \leq f(b)$
- $f$ is decreasing on $I$ if for every $a < b$ in $I$, $f(a) \geq f(b)$

Q7. $f(x) = -x^2$ is increasing for $x > 0$.
   A. True
   B. False
Let $f(x)$ be a continuous on $[a, b]$ and differentiable on $(a, b)$. There exists a value $c$ in $(a, b)$, such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$
Mean Value Theorem (not for Math 180)

Idea:

\[ f'(c) > 0 \Rightarrow \frac{f(b) - f(a)}{b - a} > 0 \Rightarrow f(b) > f(a) \]
Increasing and decreasing functions

- If \( f'(c) < 0 \) for all \( c \) in an interval \( I \), then \( f \) is decreasing on the interval \( I \).
- If \( f'(c) > 0 \) for all \( c \) in an interval \( I \), then \( f \) is increasing on the interval \( I \).
Increasing and decreasing functions

Q8. If $f'(c) = 0$ on an interval $I$, then

A. $f$ is increasing on $I$
B. $f$ is decreasing on $I$
C. $f$ is constant on $I$
D. no conclusion
Maxima and minima

\[ V(r) \]

\[ \lim_{r \to 0^+} V(r) = \infty \]

\[ r \text{ intercept} \]

\[ \lim_{r \to \infty} V(r) = 0 \]

\[ V'(R) = 0 \]

- \( V(r) \) has a **local minimum** at \( r = R \).
  - decreasing to constant to increasing
  - \( V'(r) < 0 \) to \( V'(R) = 0 \) to \( V'(r) > 0 \)

- \( V(r) \) has a **global minimum** at \( r = R \) (aka absolute minimum)
Maxima and minima

- A point $a$ is a **local minimum** of a function $f(x)$ provided that $f(x) > f(a)$ for all $x \neq a$ on an interval around $a$.

- A point $a$ is a **global minimum** of a function $f(x)$ provided that $f(x) > f(a)$ for all $x$ where $f(x)$ is defined.

- Similar definitions for a local maximum and global maximum.
Minima & Maxima

Local minima:

\[ f(x) = |x| \]

\[ f(a) \text{ is smaller than } f(x) \text{ for } x \text{ near } a. \]

\[ x = 0 \text{ local minimum! is also a global minimum.} \]
How to find maxima and minima?

Intervals where $f$ is increasing and decreasing are separated by:

- Critical points: where $f'(x) = 0$
- Singular points: where $f'(x)$ is not defined

Example: $f(x) = x^4 - 6x^3 = x^3(x - 6)$ has

$$f'(x) = 4x^3 - 18x^2 = 2x^2(2x - 9).$$

So $f'(x) = 0$ when $x = 0, x = 9/2$. 
Critical Points

\[ f'(x) = 0 \]

Increasing:
\[ f'(x) > 0 \]

Decreasing:
\[ f'(x) < 0 \]

Singular Points

\[ f(x) = \frac{1}{x} \]

Decreasing:
\[ f'(x) = -\frac{1}{x^2} \]

Not defined at \( x = 0 \).
How to find maxima and minima?

Example: \(f(x) = x^4 - 6x^3 = x^3(x - 6)\) has

\[f'(x) = 4x^3 - 18x^2 = 2x^2(2x - 9).\]

So \(f'(x) = 0\) when \(x = 0, x = 9/2\).

- \(x < 0\): \(x^2 > 0\) but \((2x - 9) < 0\) so \(f'(x) < 0\)
- \(0 < x < 9/2\): \(x^2 > 0\) but \(2x - 9 < 0\) so \(f'(x) > 0\)
- \(x > 9/2\): \(x^2 > 0\) but \(2x - 9 > 0\) so \(f'(x) > 0\)
How to find maxima and minima?

Strategy: summarize all of this in a table

<table>
<thead>
<tr>
<th>interval</th>
<th>$(-\infty, 0)$</th>
<th>0</th>
<th>$(0, 9/2)$</th>
<th>$9/2$</th>
<th>$(9/2, \infty)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'(x)$</td>
<td>negative</td>
<td>0</td>
<td>negative</td>
<td>0</td>
<td>positive</td>
</tr>
<tr>
<td></td>
<td>decreasing</td>
<td>horizontal tangent</td>
<td>decreasing</td>
<td>minimum</td>
<td>increasing</td>
</tr>
</tbody>
</table>

Since the derivative changes sign from negative to positive at the critical point $x = 9/2$, this point is a minimum. Its $y$-value is $y = f(9/2) = -\frac{3\sqrt{6}}{2}$.

On the other hand, at $x = 0$ the derivative does not change sign; while this point has a horizontal tangent line it is not a minimum or maximum.

Putting this information together we arrive at a quite reasonable sketch.

To improve upon this further we will examine the second derivative.

Example 3.6.2

The second derivative $f''(x)$ tells us the rate at which the derivative changes. Perhaps the easiest way to understand how to interpret the sign of the second derivative is to think about what it implies about the slope of the tangent line to the graph of the function.
Where are the local and global extrema?

All of these points are local maxima or local minima.

Local min
Local max
Global min
Global max.

Not a global max?
Sketching example: the Morse Potential

The Morse Potential is a refinement of the Lennard-Jones potential:

\[
W(r) = A \left( 1 - e^{-B(r-c)} \right)^2 - A.
\]

Sketch the curve and determine what the parameters \( A \) and \( B \) represent (doc cam).

1. Domain
2. Intercepts
3. Asymptotes
4. Intervals of increase and decrease, extrema
5. (next week) concavity

We’ll do this next class...
Reflection

I think the most important thing I learned this class is

A. about intervals of increase and decrease
B. the difference between local and global extrema
C. to summarize information about a function in a table before sketching
D. that I can understand complicated functions and their parameters by sketching
Reflection

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D. that I can understand complicated functions and their parameters by sketching