Announcements

- Related Rates (10 question) webwork due Oct 31
- Midterm info:
  www.math.ubc.ca/~zmurchok/180.html
  LSK 200, 6 pm, Wednesday, Oct 25
- Office Hours cancelled: Friday, Oct 20, 27

Today...

1. Related Rates Continued (2 examples)
2. Mock Midterm + Group Study
Survey

I have
A. started studying
B. started related rates webwork
C. started both
D. started neither
Survey

I am

A. doing problems
B. reading my notes
C. making study notes
D. doing problems and making note of what I can and cannot do
Survey

I am

A. doing problems
B. reading my notes
C. making study notes
D. doing problems and making note of what I can and cannot do
Types of Related Rates Problems

- **Pythagorean Theorem** (ships, things moving on orthogonal tracks)
- Areas and volumes (circular ripples, spherical balloons)
- Similar Triangles (cones, shadows)
- Trigonometric Functions (ladders, angles)
- Abstract problems (Boyle’s Law)
"RECIPE" FOR RELATED RATES

1. Draw/sketch if possible; give names to variables.
2. Wrote down relevant info from problem.
3. What do you desire? (want to find).
4. Related the variables \( z^2 = x^2 + y^2 \).
5. Reduce the problem to one-variable.
6. Differentiate with respect to time.
7. Solve actual problem (sub. in \& algebra).
8. Reality check.
Moving a camera

- A camera is placed on a tripod 2 m from the side of a road. The camera is to turn to track a car that is to drive by at 100 km/h for a video. How fast must the camera be able to turn to track the car?
Camera-road problem:

- Let $x(t)$ be the distance from the car to the camera. Let $\theta(t)$ be the angle as above.
- $\frac{dx}{dt} = -100 \text{ km/h}$. (distance $x$ is shrinking).

We want to know the rate of change of the angle the camera makes from a line perpendicular to the road, when the car passes the camera. (Maximum rotational speed)
2. Relate $x$ & $\theta$:

\[ \tan \theta = \frac{x}{0.002} \]

\[ \frac{d}{dt} \sec^2(\theta) \cdot \frac{d\theta}{dt} = \frac{1}{0.002} \frac{dx}{dt} \]

Solve:

\[ \frac{d\theta}{dt} = \frac{1}{\sec^2 \theta} \cdot \frac{1}{\frac{1}{1000}} \frac{dx}{dt} \]

\[ \frac{d\theta}{dt} = \frac{1}{(\cos^2 \theta)} \cdot \frac{1}{1000} \cdot \frac{dx}{dt} \]

\[ \Rightarrow \frac{d\theta}{dt} = \frac{500 \cos^2 \theta}{\frac{dx}{dt}} \]
We want to find:

\[
\frac{d\theta}{dt} \bigg|_{\theta = 0} = 500 \cdot \cos^2(0) \cdot (-100).
\]

Evaluate \(\frac{d\theta}{dt} \) at \(\theta = 0\):

\[
= -50000 \text{ rad/see. h.}
\]

• Check: \(\frac{d\theta}{dt} \bigg|_{\theta = 0} < 0\). Yay! \(\frac{d\theta}{dt}\) is negative, which means \(\theta\) is getting smaller.

\(\frac{d\theta}{dt} = 500 \cos^2 \theta \frac{dx}{dt}\) is maximized when \(\theta = 0\),

this matches our intuition.
A boat leaves an island and heads north at 10 km/h. A second boat leaves an island that is 30 km west of the other island, and heads south at 15 km/h. At what rate is the distance between the boats increasing when the boats are 50 km apart?
Boats:

- Let \( x(t) \) = distance travelled by boat 1.
  \[
  \frac{dx}{dt} = 10 \text{ km/h}.
  \]
- Let \( y(t) \) = distance travelled by boat 2.
  \[
  \frac{dy}{dt} = 15 \text{ km/h}.
  \]
- Let \( z(t) \) = distance between the boats.

- We want: \( \frac{dz}{dt} \) when \( z(t) = 50 \text{ km} \).

- How are \( x, y & z \) related? (A: working, B: done, C: stuck).
\[
(x+y)^2 + 30^2 = 2^2. \quad (\text{Pythagoras})
\]

\[
\frac{d}{dt} : 2(x+y) \left( \frac{dx}{dt} + \frac{dy}{dt} \right) = 2z \frac{dz}{dt}.
\]

To solve for \( \frac{dz}{dt} \) when \( z = 50 \): we need \( x+y \).

\[
\begin{align*}
40 &= x+y \quad \text{(since } 30^2 + 40^2 = 50^2) \\
\frac{dz}{dt} &= \frac{2(40)(10+15)}{2 \cdot 50} = \frac{4 \cdot 25}{5} = 20 \, \text{Km/h}
\end{align*}
\]
Muskrat spread (we didn’t do this in class)

- Square root of occupied area increases at a constant rate $k$.
Muskrat spread (we didn’t do this in class)

- In a classical paper in ecology, it was shown by the scientist Skellam (1951) that the square root of the area occupied by muskrats increased at a constant rate, $k$. 
- Determine the rate of change of the distance (from the site of release) that the muskrats had spread. Assume that the expanding area of occupation is circular.
Reflection:

1. What I find easy about related rates is the algebra.

2. “— hard ——” is setting up the problem.

3. What I need to focus on the most for the exam is
   - breathing! sleeping! algebra...
   - trig?
   - related rates?
   - tangent lines?