Announcements

- A3 (written only 1 question) and WeBWorK due Oct 3
- WeBWorK: 30 questions, graded out of 20
- Q3 on Oct 3 (first 10 minutes of class)
  - Similar to one of WeBWorK problems:
    7, 17, 18, 19, 21, 22, 23, 24, 25, 26
- Office Hours cancelled today (moved to Friday 4:30–5:30 or 5:45 in LSK 300)

Today...

1. Assignments due now & Quiz
2. Average velocity (a connection to physics)
3. Differentiation practice
Zebrafish development


- Posterior lateral line primordium migration: https://youtu.be/IqUs29Kz3HE
Zebrafish development

Figure modified from Valdivia et al. *Development* 2011.

Q1. Over time the cell cluster is
   A. speeding up
   B. slowing down
   C. moving at the same speed
Zebrafish development

<table>
<thead>
<tr>
<th>time $t$ (hours)</th>
<th>position $y$ ($\mu$m)</th>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>2</td>
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Q2. Suppose the cluster moves at a constant velocity. How fast should the cluster move to end up at the same position after 8 hours?

A. $500 \, \mu m/h$
B. $500/8 \, \mu m/h$
C. $500 \cdot 8 \, \mu m/h$
D. $8/500 \, \mu m/h$

$$v_{avg} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$= \frac{500 - 0}{8 - 0} = \frac{500}{8}$$
Zebrafish development

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C. $500 \cdot 8 \ \mu$m/h  
D. $8/500 \ \mu$m/h

\[
\begin{align*}
v_{\text{avg}} &= \frac{s(t_2) - s(t_1)}{t_2 - t_1} \\
&= \frac{500 - 0}{8 - 0} = \frac{500}{8}
\end{align*}
\]
Zebrafish development

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To get the instantaneous velocity, take a limit:

$$v = \lim_{t_2 \to t_1} v_{\text{avg}} = \lim_{t_2 \to t_2} \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

- Velocity is the derivative of the position function!
\[
\frac{s(t_2) - s(t_1)}{t_2 - t_1} = \text{slope of secant} \\
= \text{average velocity} \\
\]

\[
v = \lim_{{t_2 \to t_1}} \frac{s(t_2) - s(t_1)}{t_2 - t_1} \\
= s'(t_1). \\
\]

\text{instantaneous velocity} \\
= \text{derivative of position}
Differentiation toolbox

- \( f'(x) = \frac{df}{dx} \) same meaning
- \( \frac{d}{dx} \) (stuff) means: find the derivative of all that stuff
- Our goal: Find \( f'(x) \) for functions like

\[
f(x) = \frac{(3x^7 + 4x^2 - \frac{1}{x}) \left( x^2 + 3 \right)}{x + \sqrt{x}}
\]
Differentiation toolbox

Linearity: \[ \frac{d}{dx} (af(x) + bg(x)) = a\frac{df}{dx} + b\frac{dg}{dx} \]

Power Rule: \[ \frac{d}{dx} (x^n) = nx^{n-1}, \quad n \neq 0 \]

Q3. If \( f(x) = \frac{8}{x^8} \), then \( f'(x) \) is

A. \( \frac{1}{x^9} \)
B. \( -\frac{1}{x^9} \)
C. \( \frac{64}{x^9} \)
D. \( -\frac{64}{x^9} \)
Q3. \[ f'(x) = \frac{8}{x^8} = 8x^{-8} \] (Power Rule)

\[ f'(x) = 8 \cdot (-8) x^{-8-1} \]

\[ = -64x^{-9} \]

\[ = -\frac{64}{x^9} \]

\[ \frac{d}{dx} \left( \frac{HI}{Low} \right) = \frac{\text{Low dHI} - \text{HI dLow}}{\text{Low}^2} \]
Differentiation toolbox

Product Rule: \[
\frac{d}{dx} (fg) = \frac{df}{dx}g + f \frac{dg}{dx}
\]

Quotient Rule: \[
\frac{d}{dx} \left( \frac{p}{q} \right) = \frac{q \frac{dp}{dx} - p \frac{dq}{dx}}{q^2}
\]

Q4. If \( f(x) = (3x^7 + 4x^2)(x^2 + 3) \), then \( f'(x) \) is ... not a clicker question ... you do it!
Q1: \( f(x) = (3x^7 + 4x^2)(x^2 + 3) \).

\[ f'(x) = (21x^6 + 8x)(x^2 + 3) + (3x^7 + 4x^2)(2x) \]

\[ \frac{d}{dx} (x^7) = 7x^6 \]
\[ \frac{d}{dx} (3x^7) = 3 \frac{d}{dx} (x^7) = 3 \cdot 7x^6 = 21x^6. \]

\[ f(x) = (3x^7 + 4x^2)(x^2 + 3) \]
\[ = 3x^9 + 4x^4 + 9x^7 + 12x^2 \]

\[ f'(x) = 27x^8 + 16x^3 + 63x^6 + 24x. \]
Q5: \[ f(x) = \frac{(3x^7 + 4x^2)(x^2 + 3) - (x + \frac{1}{x})}{x + \sqrt[3]{x}} \]

Quotient:
(BAD BECAUSE there is a product in the numerator)

\[ f'(x) = \frac{(x + \sqrt[3]{x}) \left[ (3x^7 + 4x^2)(x^2 + 3) - (x + \frac{1}{x}) \right]'}{(x + \sqrt[3]{x})^2}. \]

\[ \left[ (3x^7 + 4x^2)(x^2 + 3) - (x + \frac{1}{x}) \right]' = \left[ (3x^7 + 4x^2)(x^2 + 3) \right]' - \left[ (x + \frac{1}{x}) \right]'. \]

\[ = 21x^6 + 16x^3 + 63x^6 + 24x = \left[ 1 - \frac{1}{x^2} \right]. \]

\[ = 1 + x^{-2} \]

\[ \frac{d}{dx} \left( \frac{1}{x} \right) = \frac{d}{dx}(x^{-1}) = -1x^{-2} = \frac{-1}{x^2}. \]
\[(x + \sqrt{x})' = 1 + \frac{1}{2\sqrt{x}} \quad \text{and} \quad \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{\frac{1}{2}})\]

\[
\begin{align*}
\frac{d}{dx}(\sqrt{x}) &= \frac{1}{2} \cdot x^{-\frac{1}{2}} - 1 \\
&= \frac{1}{2} \cdot x^{-\frac{1}{2}} \\
&= \frac{1}{2 \sqrt{x}}
\end{align*}
\]

\[f'(x) = (x + \sqrt{x}) \left[ 27x^8 + 16x^3 + 63x^6 + 24x - 1 + \frac{1}{x^2} \right] - \left[ (3x^7 + 4x^2)(x^2 + 3) - (x + \frac{1}{x}) \right] \left( 1 + \frac{1}{2\sqrt{x}} \right)\]

\[(x + \sqrt{x})^2 \]
Your turn: find \( f'(x) \) for \( f(x) = \frac{x^5 - 5x^4}{x} \).

Quotient Rule:

\[
f'(x) = \frac{x \cdot (5x^4 - 20x^3) - (x^5 - 5x^4) \cdot 1}{x^2}
\]

Simplifying:

\[
f'(x) = \frac{x^5 - 5x^4}{x} = \frac{x^5}{x} - \frac{5x^4}{x} = x^4 - 5x^3
\]

\[
f'(x) = 4x^3 - 15x^2.
\]
Reflection

In a box in your notes,

1. write down one thing you need to practice more
2. write down one question that you have.