Hello!

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Office Hours: T 12–13, W 11–13 in LSK 300

Upcoming:

Diagnostic Test
A1 (written and WeBWorK) due Sep 19
WeBWorK: 30 questions, graded out of 20
Q1 on Sep 19 (first 10 minutes of class)
  Similar to WeBWorK problems:
    11, 13, 14, 16, 23, 25, 26, 27, 28, 29.

Workshops start today
50 minutes group work with submitted work
30 minutes study hall (= support)
Last Time: Lennard-Jones Potential

- The Lennard-Jones Potential,

\[ V(r) = \left( \frac{1}{r} \right)^{12} - 2 \left( \frac{1}{r} \right)^6, \]

describes the interaction between a pair of atoms at a distance \( r \) apart.

- What does this function look like?
  - Domain
  - Zeros
  - Asymptotes

- Today: Horizontal and Vertical Asymptotes, Limits.
Horizontal Asymptotes = Limits at ±∞

Example

Find the horizontal asymptotes of \( f(x) = \frac{1}{(x+3)^2} \).

- As \( x \to \infty \), \((x + 3)^2 \to \infty\), hence
  \[
  \lim_{x \to \infty} \frac{1}{(x + 3)^2} = 0.
  \]

- \( y = 0 \) is a horizontal asymptote of \( f(x) \).

- www.desmos.com/calculator
Lennard-Jones Potential: Asymptotes?

Q1. Find the horizontal asymptotes of

\[ V(r) = \left( \frac{1}{r} \right)^{12} - 2 \left( \frac{1}{r} \right)^6 \]

A. There are none
B. \( x = 0 \)
C. \( y = 0 \)
D. I don’t know
\[ v(r) = \left( \frac{1}{r} \right)^{12} - 2 \left( \frac{1}{r} \right)^6. \]

\[ \lim_{r \to \infty} \left( \frac{1}{r} \right)^{12} - 2 \left( \frac{1}{r} \right)^6 = 0^{12} - 2(0)^6 = 0. \]

\[ \text{Since the limits at } \pm \infty \text{ are both zero, there is a H.A. at } y = 0. \]
\[ \text{Ex. } \lim_{x \to \infty} \frac{\sqrt{2x^2 - 1}}{x + 1}. \]

\begin{align*}
\text{Dominant powers} \\
\text{As } x \to \infty, \quad & x + 1 \text{ approx } x \quad . \\
& \sqrt{2x^2 - 1} \text{ approx } \sqrt{2x^2} \\
\lim_{x \to \infty} \frac{\sqrt{2x^2 - 1}}{x + 1} &= \lim_{x \to \infty} \frac{\sqrt{2x^2}}{x} \\
&= \lim_{x \to \infty} \frac{\sqrt{2}x}{x} \\
&= \sqrt{2} \\
\Rightarrow \text{H.A at } y = \sqrt{2} \\
\text{(how to evaluate limits at } \infty). \]
\end{align*}

\begin{align*}
\text{Algebraic Method} \\
\Rightarrow \text{Factoring out the dominant powers}. \\
\lim_{x \to \infty} \frac{\sqrt{2x^2 - 1}}{x + 1} \\
&= \lim_{x \to \infty} \frac{\sqrt{x^2(2 - \frac{1}{x^2})}}{x} \\
&= \lim_{x \to \infty} \frac{x(1 + \frac{1}{x})}{x(1 + \frac{1}{x})} \\
&= \frac{\sqrt{2 - 0}}{1 + 0} = \sqrt{2}. \\
\end{align*}
Horizontal Asymptotes

Q2. Find the following limit.

$$\lim_{x \to \infty} \frac{2x^2 - 1}{(x + 3)^2}$$

How confident in your answer are you?

A. Very confident
B. Confident
C. Neutral
D. I don’t know
E. I don’t know at all
Horizontal Asymptotes

Q3. Find the following limit.

\[ \lim_{x \to -\infty} \frac{2x^2 - 1}{(x + 3)^2}. \]

How confident in your answer are you?

A. Very confident
B. Confident
C. Neutral
D. I don’t know
E. I don’t know at all
Q2. Q3. Hacking

\[ \lim_{x \to \infty} \frac{2x^2 - 1}{(x+3)^2} = \lim_{x \to \infty} \frac{2x^2}{x^2} \]

Algebraically

\[ \lim_{x \to \infty} \frac{2x^2 - 1}{x^2 + 6x + 9} = \lim_{x \to \infty} \frac{x^2(2 - \frac{1}{x^2})}{x^2(1 + \frac{6x}{x^2} + \frac{9}{x^2})} = \frac{2 - 0}{1 + 0 + 0} = 2 \]

The same occurs as \( x \to -\infty \).

\[ (x + 3)^2 = (x+3)(x+3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9. \]

Example:

\[ \lim_{x \to -\infty} \frac{\sqrt{2x^2 - 1}}{x + 1} \]

This means \( x < 0 \)

\[ = \lim_{x \to -\infty} \frac{\sqrt{x^2(2 - \frac{1}{x^2})}}{x(1 + \frac{1}{x})} = \lim_{x \to -\infty} \frac{\sqrt{2 - \frac{1}{x^2}}}{1 + \frac{1}{x}} \]

\[ = \lim_{x \to -\infty} \frac{-\sqrt{2 - \frac{1}{x^2}}}{1 + \frac{1}{x}} = \frac{-\sqrt{2}}{1} = -\sqrt{2} \]

\[ |x| = \begin{cases} x, & x > 0, \\ -x, & x < 0. \end{cases} \]

(piecewise function)
Horizontal Asymptotes

- To find horizontal asymptotes of \( f(x) \)...
calculate

\[
\lim_{x \to \infty} f(x) \quad \text{and} \quad \lim_{x \to -\infty} f(x).
\]

- Look at the dominant powers, or factor out the dominant powers for an algebraic calculation.
- When looking at \( x \to -\infty \)... watch out for negative factors!
But what about other limits and vertical asymptotes?
Lennard-Jones Potential: Limits

\[ V(r) = \left( \frac{1}{r} \right)^{12} - 2 \left( \frac{1}{r} \right)^{6} \]

Q4. Find the following limit.

\[ \lim_{r \to 2} V(r). \]

A. The limit does not exist.
B. \( \left( \frac{1}{2} \right)^{12} - 2 \left( \frac{1}{2} \right)^{6} \)
C. \( y = 0 \)
D. I don’t know.
\[ f(x) = \frac{1}{x} \]

- \[
\lim_{{x \to 2^-}} f(x) = \lim_{{x \to 2^-}} \frac{1}{x} = \frac{1}{2}.
\]

\( \text{limit from the left:} \)
\( x \to 2^- \)
\( \text{means } x \text{ goes to } 2 \)
\( \text{but } x < 2 \)

- \[
\lim_{{x \to 2^+}} f(x) = \lim_{{x \to 2^+}} \frac{1}{x} = \frac{1}{2}.
\]

\( \text{limit from the right:} \)
\( x \to 2^+: \) \( x \text{ goes to } 2, \text{ but } x > 2 \)

Since the left-sided and right-sided limits agree, we say the limit exists and write
\[
\lim_{{x \to 2}} \frac{1}{x} = \frac{1}{2}.
\]
\( f(x) = \frac{1}{x} \) also has **vertical asymptote** (Hint: the domain is \( x \neq 0 \)). Let's check:

- \( \lim_{x \to 0^+} \frac{1}{x} = +\infty \)
- \( x > 0 \)
- \( \frac{1}{x} > 0 \)

Graphically:

We say that \( f(x) = \frac{1}{x} \) has a vertical asymptote at \( x = 0 \).
Q5. If \( f(x) = \frac{1}{(x-3)(x-2)} \), find
\[
\lim_{x \to 2^-} f(x) \quad \text{and} \quad \lim_{x \to 2^+} f(x).
\]

A. \(+\infty, -\infty\)
B. \(-\infty, +\infty\)
C. \(+\infty, +\infty\)
D. \(-\infty, -\infty\)
\[ f(x) = \frac{1}{(x-3)(x-2)} \]

\[ \lim_{x \to 2^-} \frac{1}{(x-3)(x-2)} = +\infty \]

* Keep track of signs of factors!

Ex: \[ \lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \frac{f(3)}{f(3)} \]

\[ \lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)} = \lim_{x \to 3} (x+3) = 6 \]

No limit exists, but \( x \neq 3 \) is the domain of \( f(x) \).
Mini-quiz: Do I get it?

\[ f(5) = 2 \]

\[ \lim_{x \to 0} f(x) = 0 \]

\[ \lim_{x \to 5} f(x) \text{ D.N.E.} \]

\[ \lim_{x \to 5^{-}} f(x) = 2, \text{ but } \lim_{x \to 5^{+}} f(x) = 1. \]

VA$_s$? \[ x = 10 \]

HA$_s$? \[ y = 1 \text{ as } x \to \infty. \]
Today...

- Horizontal Asymptotes
- Vertical Asymptotes
- Thinking about dominant powers helps calculate limits

- Limits
  - $x \to a$
  - Right-sided limits $x \to a^+$
  - Left-sided limits $x \to a^-$